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A Hybrid System Approach to Power Systems Voltage Control

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***Abstract*— Emergency voltage control problems in electric power networks, which typically feature relatively slow dynamics, have stimulated the interest for the implementation of online optimal control techniques. Nonetheless for realistically sized networks the complexity of the associated optimization problem may prove to be prohibitive; in this sense it is therefore necessary to provide an adequate modelling scheme that captures the interdependence between discrete and continuous dynamics and the behaviour of the inherent nonlinearities while yielding a system description that is not overly onerous and viable for online computation purposes. In view of this objective the present paper extends previous work done in terms of a model predictive voltage control approach in power systems based on a mixed-logical-dynamical model formulation while employing an alternative and simpler scheme to extend the applicability of the method to a 12-bus sample network model.**

I. INTRODUCTION

Over the last decade we have witnessed a worldwide trend in the restructuring and privatization of electrical power systems that has brought about an accrued interest in alternative forms of organization, design and operation of power networks. In particular, there is an increased economical interest in exploiting the system characteristics so as to operate it closer to its physical limits. At the same time, due to environmental reasons, it is usually difficult to expand the network by means of new transmission lines. All this accounts for the fact that nowadays electrical networks are placed under greater strain than before. Therefore the risk of malfunctioning or system collapse has increased, as also confirmed by the recent large scale blackouts throughout the world [1]. The present situation calls for appropriate countermeasures to be taken in the direction of superseding still commonly used, local control schemes (such as undervoltage load shedding) with schemes capable of capturing the complexity of the global system dynamics.

In this sense in the recent past we have seen considerable interest arising in optimal control schemes for voltage stability problems, since the voltage decay in power grids often features relatively slow dynamics - from several seconds to a few minutes - thus allowing the necessary online optimization procedure to take place and be applied so that voltage collapse can be averted.

An electrical power network is typically a considerably articulate, nonlinear system involving a number of heterogeneous and diversified elements whose mathematical description requires the use of both continuous and discrete formalisms. Recent research work has correspondingly proposed a diversified set of solutions; in [2],[3] a linearized model of the network is employed and the model predictive control (MPC) problem solved by means of a heuristic tree search algorithm to determine the control values to be applied to the system. The approach used in [4] employs a non linear predictive controller featuring a reduced-complexity parameterization of the open loop control sequence that permits the online solution within adequate computation times for the considered model. A method based on the trajectory sensitivity approach [5] is used in [6], [7] to stabilize a power network with an open loop control sequence. In [8], a detailed representation of the considered network and its nonlinearities is considered and formulated in terms of a mixed-logical-dynamical (MLD) model based on which an MPC control problem is formulated and solved. The present work also employs an MLD/MPC based method, using however an alternative formulation that exploits a simpler, but sufficiently accurate, description of the system characteristics. This allows one to consider a sample network which, although still smaller than a realistic network of industrial interest, is nonetheless of increased size and complexity.

This paper is organized as follows. In Section II, an overview of the considered sample network and of the modelling approach is given and formalized in the MLD framework. In Section III, an optimal control problem featuring the appropriate control objectives is formulated. Section IV contains simulation results illustrating the control scheme's performance. Finally, conclusions and further research directions are outlined in Section V.

II. SYSTEM NETWORK AND MODEL

The case study in consideration has been proposed by ABB as a benchmark in the framework of the European Project *Control and Computation*. The interested reader is referred to the related report [9] for a complete description of the system. In the following subsections the network structure, its components and the chosen modelling approach are reviewed. All quantities are given as per unit (p.u.) values.

A. Network Topology

The considered power system contains the three areas A, B, C, see Fig. 1. For simplicity, the three areas are identical with the exception of certain parameter values. Moreover the generator in area A represents an infinite bus (i.e. the related bus has constant voltage), whereas the two generators in areas B and C are modelled as physical machines of limited capacity. The three areas are connected with three lossless double tie lines L_{AB},L_{AC} and L_{BC} modelling a

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transmission network, of reactance value 1.6, 0.8 and 0.8 respectively. The outages of these lines will be considered as the (measured) disturbance in the simulations featured in Section IV. Each area is a meshed sub-transmission system consisting of three lossless lines featuring a transformer with an on-load tap changer (OLTC) controller and a capacitor bank. The sub-transmission systems feed the distribution networks that have been aggregated into a single load model, featuring a load shedding control input [9]. In the following the index $i = \{A, B, C\}$ is employed to refer to the three areas, and $h = \{1, 2, 3, 4\}$ to the buses within each of these. Voltage magnitudes are denoted by a subscript indicating the network bus, i.e. v_{A1} is the voltage of bus $A1$.

B. Network Components

In the following a concise description of the system elements and inputs (OLTC, capacitor bank, load shedding controls) is given.

1) Physical generator: The physical generator consists of a synchronous machine with an automatic voltage regulator (AVR). The transient and subtransient characteristics of the synchronous machine are not modelled as the time scale of the considered voltage stability phenomena is considerably longer, hence a static model of the synchronous machine is used here. In particular, the generators of areas B and C can be reduced [8] to a single nonlinear expression for the field voltage $e_i := e_i(v_{i2}, Q_{iG})$ where Q_{iG} represents the reactive power generated by the machine. Saturation is included in the AVR to account for the maximum allowable current in the excitation system, that is e_i has an upper limit value E_{max}; once a machine has reached its saturation limit it cannot produce additional reactive power and can therefore no longer participate in sustaining the voltages in the network [10].

2) Load: The aggregate load model is a nonlinear system with recovery dynamics [11] described by the following first order differential equation

$$\dot{x}_{Li} = -\frac{x_{Li}}{T} + P_0(\sqrt{v_{i4}} - (v_{i4})^2) \tag{1}$$

where x_{Li} represents the internal state of the load for each area, T=60 seconds is the power recovery time constant and P_{i0} is the active power steady state value for area i.

3) OLTC: The OLTC controller manipulates the turn ratio n_i to control the secondary voltage. The transformer connects the sub-transmission system to the aggregate load model and must therefore feature an appropriate turns ratio to accordingly step down the voltage. The ratio n_i may vary in ten discrete steps of $+/-\ 2\%$ around the central value 1, i.e. the range of possible values is $n_i \in \{0.8, 0.82, \ldots 1, \ldots 1.18, 1.2\}$. The control input is directly assumed to be n_i, thus bypassing local OLTC controllers.

4) Capacitor bank: The capacitor bank, or reactive power bank, is employed to locally stabilize the bus voltages close to the load. The available set of inputs c_i representing the quantity of reactive power injected in the grid is $c_i \in \{0, 0.2, 0.4, 0.6\}$.

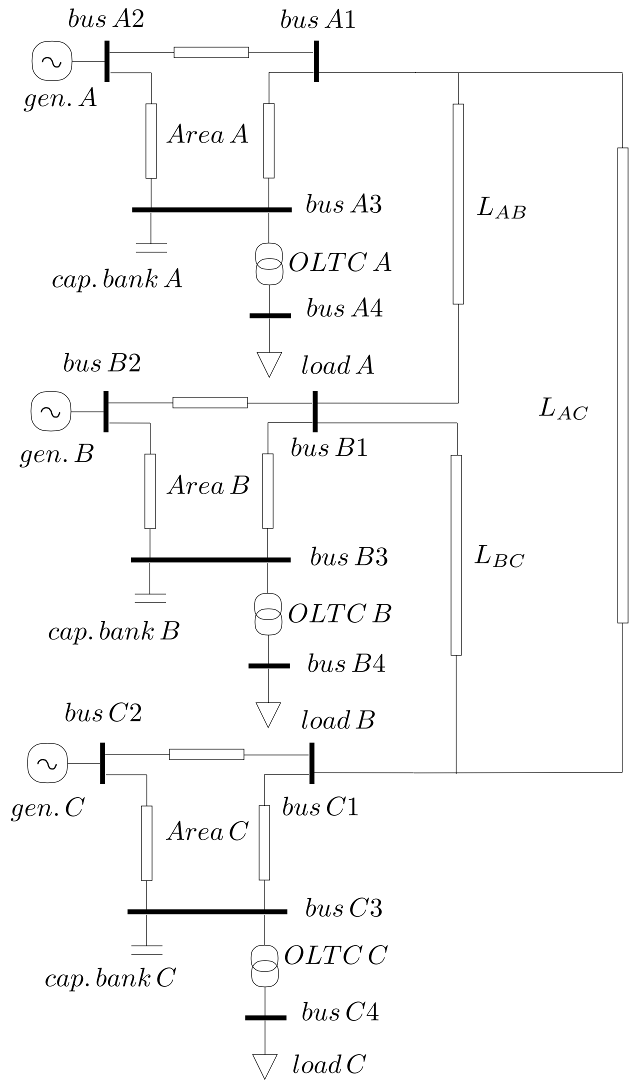


Fig. 1. Topology of the considered network

5) Load shedding: Following the occurrence of an outage, in the case that the network is excessively strained and overloaded, load can be shed in discrete steps to relieve the grid of part of the load using the the inputs $s_i \in \{0, 0.1, 0.2, 0.3\}$. For each area i the active and reactive load demands P_{iL} and Q_{iL}

$$P_{iL} = (1 - s_i)(\frac{x_{iL}}{T} + P_{i0}(v_{i4})^2) \tag{2a}$$

$$Q_{iL} = \frac{Q_{i0}}{P_{i0}} P_{iL} \tag{2b}$$

are reduced accordingly, where Q_{i0} is the reactive power steady state value for area i.

C. Network Modelling Procedure

The network system under consideration can be conveniently expressed [12], [2] in the general differential-algebraic equation (DAE) form

$$\dot{x}_{iL} = f(x_{iL}, v_{i4}) \tag{3a}$$

$$0 = g(x_{iL}, u, v_{i4}, w) \tag{3b}$$

where u denotes the system inputs. The state variables are the dynamic load variables x_{iL} appearing in (1) and as such cannot change instantaneously. On the other hand, the algebraic variables v_{i4} (denoting the load voltage magnitudes) and w (denoting the voltages of all buses and the excitation fields) do not appear as derivatives, and can thus change instantaneously due to changes in u or in the network topology.

The algebraic equations (3b) consist of power balances and expressions for the excitation fields of the generators. Given the inputs to the grid and the state of the load, (3b) can be solved to yield the network and field voltages. The load voltages can be considered as inputs to the dynamical part of the system (3a) thus determining the evolution of the load states. By linearizing (3a) and (3b) at x_{iL}^*, u^* and time-discretizing the system with the sampling interval T_s it is always possible to formulate the following model in the affine expressions of $x_{iL}(k), u(k), v_{i4}(k)$ and $w(k)$

$$x_{iL}(k+1) = \hat{A}x_{iL}(k) + \hat{B}v_{i4}(k) + \hat{C} \tag{4a}$$

$$v_{i4}(k) = \hat{D}x_{iL}(k) + \hat{E}u(k) + \hat{F} \tag{4b}$$

$$w(k) = \hat{G}x_{iL}(k) + \hat{H}u(k) + \hat{I} \tag{4c}$$

wherein k denotes the discrete time instants kT_s; $\hat{A}$, $\hat{B}$, $\hat{C}$, $\hat{D}$, $\hat{E}$, $\hat{F}$, $\hat{G}$, $\hat{H}$, $\hat{I}$ are the matrices resulting from the linearization and time-discretization procedure of the dynamic and static components of the network equations. Therefore a complete, linearized, discrete time model of the system can readily be computed.

The physical generators (and thus also the system variables that more strongly depend on their mode of operation, i.e. the voltages of the buses in their vicinity) however feature a saturating nonlinear behaviour that cannot be accounted for in this manner. To devise a remedy for this, the following line of reasoning is adopted. For any given $x_{iL}(k), u(k)$ an approximate value of $w(k)$ is computed on the basis of (4c). From this the values corresponding to the excitation fields are extracted and compared to the saturation value E_{max}; if the generator in area i exceeds this limit then the corresponding values of the voltages for the buses belonging to area i, as calculated from (4b) and (4c), are reduced by a positive attenuation factor $\alpha_{ih} < 1$. This in turn affects the system update equation as the load voltages $v_{i4}(k)$ appear then in (4a).

The foregoing approach is based on the fact that during voltage decay phenomena the response of the generators is to typically increase reactive power production in an attempt to curb the monotonic decrease of voltages. When the saturation limit of the excitation field is reached the maximum reactive power production is achieved, so that the system is not able to compensate on its own for the lack of reactive power anymore. At this point voltages of buses in the proximity of saturated generators tend to exhibit an increase in their rate of decay [10] which justifies the introduction of the reduction coefficients α_{ih} to account for the related voltage magnitude diminution.

In other words, stemming from this procedure, and by appropriately combining (4a), (4b) and (4c) for the four possible modes of operation of the network, i.e. with none, both or either of the generators saturated, a piecewise affine (PWA) description of the network equations (3a) and (3b) is derived and is expressed as

$$x_{iL}(k+1) = \bar{A}_j x_{iL}(k) + \bar{B}_j u(k) + \bar{C}_j \tag{5a}$$

$$w(k) = \bar{G}_j x_{iL}(k) + \bar{H}_j u(k) + \bar{I}_j \tag{5b}$$

$$\text{if} \quad \bar{P}_j w(k) + \bar{Q}_j \leq 0, \quad j \in \{1,2,3,4\} \tag{5c}$$

D. MLD Framework

The chosen approach towards modelling the system employs an MLD formulation as it captures the associated hybrid features and allows the definition of the optimal control problem in a convenient way. Furthermore, efficient software (HYSDEL, HYbrid System Description Language) is available to easily translate a high level system description into the chosen formalism [13], and PWA [14] elements can also be treated directly and conveniently cast in an MLD representation. The general form of MLD hybrid systems as introduced in [15] is

$$x(k+1) = Ax(k) + B_1u(k) + B_2\delta(k) + B_3z(k) + B_5 \tag{6a}$$

$$y(k) = Cx(k) + D_1u(k) + D_2\delta(k) + D_3z(k) + D_5 \tag{6b}$$

$$E_2\delta(k) + E_3z(k) \leq E_1u(k) + E_4x(k) + E_5\,, \tag{6c}$$

where $x \in \mathbb{R}^{n_c} \times \{0,1\}^{n_\ell}$ denotes the states, $u \in \mathbb{R}^{m_c} \times \{0,1\}^{m_\ell}$ the inputs and $y \in \mathbb{R}^{p_c} \times \{0,1\}^{p_\ell}$ the outputs, with both real and binary components. Furthermore, $\delta \in \{0,1\}^{r_\ell}$ and $z \in \mathbb{R}^{r_c}$ represent binary and auxiliary continuous variables, respectively. These variables are introduced when translating propositional logic or PWA functions into linear inequalities. Only MLD systems that are *completely well-posed* are considered, i.e. systems for which for given $x(k)$ and $u(k)$, the values of $\delta(k)$ and $z(k)$ are uniquely determined by the inequality (6c). This assumption is not restrictive and is always satisfied when real plants are considered.

III. Optimal Control

A. Control Objectives

The primary aim of the emergency control scheme is to keep all voltages at values between 0.9 p.u and 1.1 p.u around the nominal point of operation, thus ensuring that the system is sufficiently far away from the point of collapse. The secondary aim is to minimize switching of the control inputs. In particular, load shedding is to be avoided load unless absolutely so as to fulfill the primary objective. The tertiary objective is to keep the load voltages as close as possible to the nominal value 1 p.u.. In view of the chosen sampling period T_s computational delay times of up to 30 seconds are acceptable, although control actions are naturally more beneficial if applied as soon as possible following a disturbance [9].

B. Model Predictive Control

Model Predictive Control (MPC) has been traditionally and successfully employed in the process industry and recently also for hybrid systems [15]. The control action is obtained by minimizing an objective function at each time step over a finite horizon subject to the equations and constraints of the model.

The major advantage of MPC is its straight-forward design procedure. Given a model of the system, one only needs to set up an objective function that incorporates the control objectives. Hard constraints can be easily dealt with by modelling them directly as inequality constraints, whereas soft constraints can be accounted for in the objective function by using large penalties. For details concerning the set up of the MPC formulation in connection with MLD models, the reader is referred to [15] and [16]. Further details about MPC can be found in [17].

C. Optimal Control Problem

To account for the aforementioned control objectives a cost function is formulated similarly as in [8], [2] reflecting the order of importance of the selected control criteria. Let the slack variables t_{ih} defined by

$$\begin{cases} 0.9 - v_{ih}(k) \leq t_{ih}(k) \\ -1.1 + v_{ih}(k) \leq t_{ih}(k) \\ 0 \leq t_{ih}(k) \end{cases} \tag{7}$$

denote the amount of violation of the condition imposed by the primary objective for each bus. This formulation is introduced so as to ensure that this constraint is effectively satisfied and leads to eleven slack variables (bus $A1$ has constant voltage by definition) at each sampling instant k, grouped in the vector $t(k) = [t_{ih}(k)]^T$.

Define now the switching on the manipulated variables as

$$\Delta u(k) = [\Delta n_i(k), \Delta c_i(k), \Delta s_i(k)]^T \tag{8}$$

so that Δu features nine components.

Lastly, the deviation of the load voltages from their reference values is expressed by the three component error vector

$$\varepsilon(k) = [|v_{i4}(k) - 1|]^T \tag{9}$$

Define the diagonal penalty matrices $Q_t = \text{diag}(q_{t1}, \ldots, q_{t11})$, $Q_{\Delta u} = \text{diag}(q_{\Delta u1}, \ldots, q_{\Delta u9})$ and $Q_\varepsilon = \text{diag}(q_{\varepsilon 1}, \ldots, q_{\varepsilon 3})$ with all penalty weights $\in \mathbb{R}^+$ and where the entries in Q_t, $Q_{\Delta u}$ and Q_ε are correlated to the correspondent ordering in $t(k)$, $\Delta u(k)$ and $\varepsilon(k)$. Consider the expression for the stage cost

$$S(k) = \|Q_t\, t(k)\|_\infty + \|Q_{\Delta u}\, \Delta u(k)\|_\infty + \|Q_\varepsilon\, \varepsilon(k)\|_\infty \tag{10}$$

and the formulation of the cost function

$$J(x(k), u(k-1), U(k)) = \sum_{\ell=0}^{N-1} S(k+\ell|k) \tag{11}$$

which penalizes the predicted evolution of $S(k+\ell|k)$ from time-instant k on over the finite horizon N using the ∞-norm.

The control law at time-instant k is then obtained by minimizing the objective function (11) over the sequence of control inputs $U(k) = [u(k), \ldots, u(k+N-1)]^T$ subject to the related system equations and constraints for the MLD model.

IV. Simulation Results

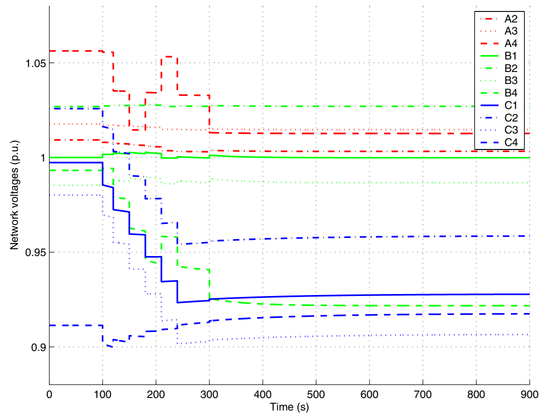
In this section, simulation results for different scenarios are given illustrating the performance of the proposed control methodology.

The penalty matrices are chosen such that a weight of 150 is placed on the violation of each soft constraint; the inputs are weighted according to their order of importance so that the penalty coefficients for Δn_i, Δc_i and Δs_i are 2, 20 and 60 respectively. Finally, a weight of 0.1 is chosen for the components of ε. The prediction horizon is $N = 4$. This results in the computation time being always a fraction of a second. At each sampling instant, the linearization point x^*, u^* is chosen by taking the current state $x(k)$ and the input applied at the preceding time instant $u(k-1)$. To this end, the HYSDEL modelling environment [13] features the possibility of giving the values for (5) in parametric form, so that the relevant numerical entries are then seamlessly inserted and updated on-line as each new linearization is performed. The parameters α_{ih} are all set to the value 0.97. The system time constant T featured in (1) is 60 seconds, and the sampling interval T_s is taken to be 30 seconds. All results shown in the following figures are normalized, and one time unit is equal to one second for the chosen time scale.

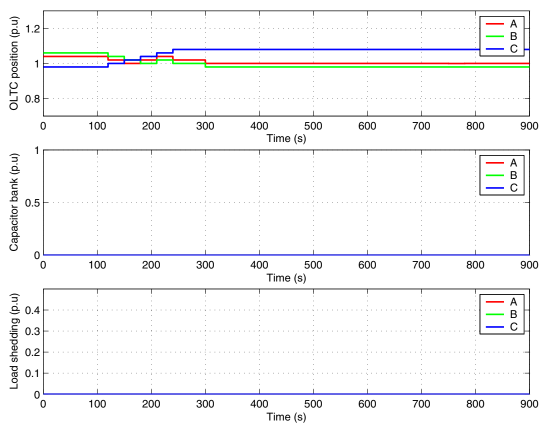
The first case concerns the partial outage of line L_{BC} at $t = 100$, during which the reactance of the transmission line increases from 0.8 to 1.6 p.u.. Fig.2(a) depicts the evolution of the network voltages and Fig.2(b) the employed control sequence. Following this disturbance the OLTCs are appropriately tapped up or down in order to maintain voltages in area C above the critical value of 0.9; for this case no additional set of controls is required to avoid the risk of exceeding this bound.

The second case to be analyzed is that of the complete outage of line L_{BC}, after which the line effectively becomes an open circuit. This leads to a situation where areas B and C are not directly connected. Fig.3(a) and Fig.3(b) depict the network voltages and the inputs; as in this case the contingency is more severe, the array of employed control actions extends to the capacitor banks, so that at instant $t = 150$ additional reactive power is injected into area C in order to increase voltage levels in that area; indeed, area C is the most exposed to risks of voltage collapse as the complete outage of line L_{BC} isolates it more than it does area B, which is connected to the strong area A through a line of smaller reactance.

Lastly, the third case to be considered features the outage of line L_{AB} at $t = 100$, after which the line is an open cir-

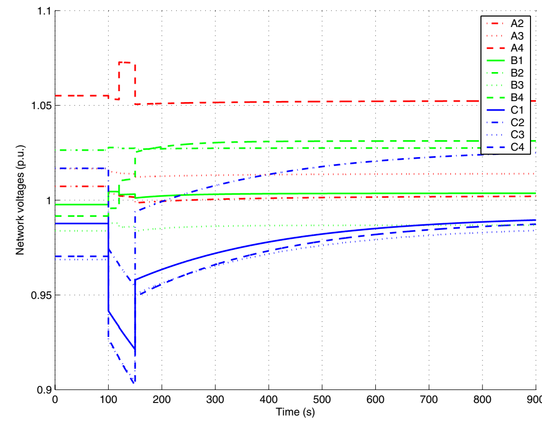


(a) Temporal evolution of network voltages

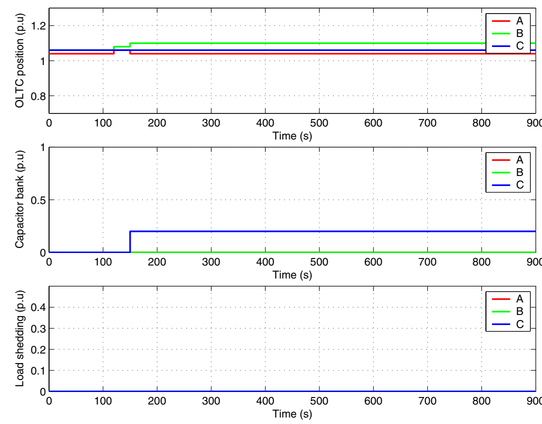


(b) Control input sequence

Fig. 2. Simulation results for the first scenario



(a) Temporal evolution of network voltages



(b) Control input sequence

Fig. 3. Simulation results for the second scenario

cuit. The related results are depicted in Fig.4(a) and Fig.4(b); for this scenario all controls must be employed. In particular load shedding is also applied immediately following the occurrence of the outage at $t = 120$ to compensate for the critically low voltage levels (< 0.9); successive tapping on the OLTCs tries to keep all voltage magnitudes within bounds, until at $t = 570$ the capacitor bank in area C is switched to sustain the bus voltages in area A (areas A and B are not directly connected) as well as its own.

V. Conclusions and Outlook

An alternative formulation of the voltage control problem in electrical power networks has been presented. The MLD framework has been used to model the hybrid dynamics of a sample benchmark 12-bus network. On the basis of the obtained model an optimal control problem has been formulated and solved online. The use of MPC allows one to explicitly account for the hard physical constraints on the input variables and to effectively stabilize the system voltages for a variety of test cases featuring different operating points and line outages. The desired behaviour can be achieved by numerically tuning the selected cost function on the basis of the required control objectives.

The present work extends previous work done in this area so that the size of tractable systems could be effectively increased to consider networks of higher dimension and complexity. While this represents one step towards a more realistic setup, electrical grids are still significantly larger and the need to take this into account must accordingly be addressed.

VI. Acknowledgements

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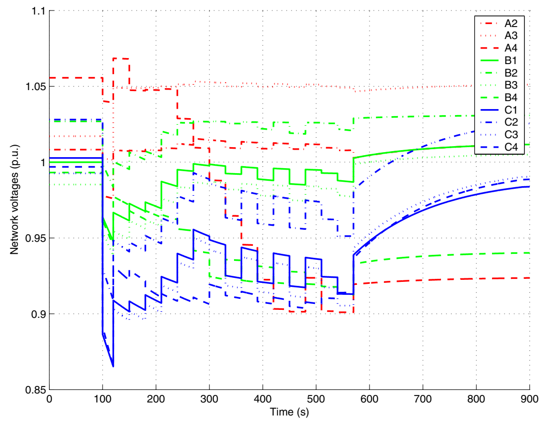
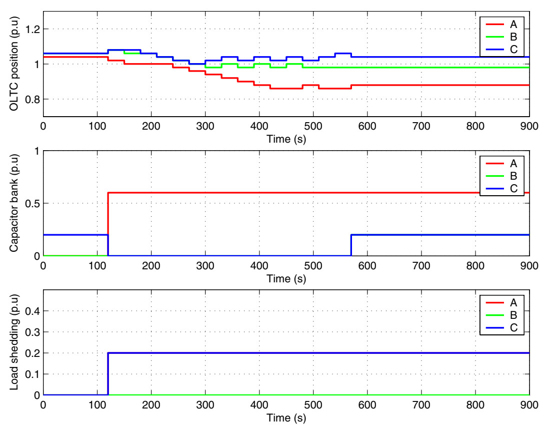


(a) Temporal evolution of network voltages



(b) Control input sequence

Fig. 4. Simulation results for the third scenario

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