Supervisory Water Level Control for Cascaded River Power Plants

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ABSTRACT

The discharge through the facilities of a river power plant is adjusted to control the water level at a pre-specified point. The common control method often yields large unnatural discharge variations resulting in unsatisfactory control performance. In cascades of river power plants, these discharge variations are unpredictably amplified affecting adversely nature and imposing problems on navigation.

As a solution to this problem, this paper presents a supervisory controller for cascaded river power plants, which is based on Model Predictive Control. The required linear discrete time model of the power plant cascade is derived from the Saint Venant equations. The objective of the controller is to keep the pre-specified water levels within given bounds and to dampen the discharge variations. This is expressed in a quadratic cost function subject to constraints. A Kalman filter is used to estimate the current values of the state variables from the available water level measurements.

The main advantages of the proposed control scheme are a coordination of the control actions for the whole cascade taking interactions between the power plants into account, preemptive control for anticipated disturbances, explicit constraint handling, and straightforward tuning. The proposed concept is compared with the currently employed PI-type controllers via simulations, demonstrating the achieved enhancements. In particular, the damping of disturbances is significantly improved while the water level constraints are met.

Keywords - *Model Predictive Control, Constrained Optimal Control, Cascaded River Power Plants, Saint Venant Equations, River Modelling*

I INTRODUCTION

River power plants are man-made constructions, which are built into the course of a river to generate electrical energy. These facilities have a major impact on the water level and the flow of the river as they retain water and vary the discharge through their facilities, often inducing unnatural water level and discharge variations. To limit the adverse impact on nature as much as possible, the authorities impose conditions on the operation of power plants. These conditions usually imply keeping the so-called concession level, the water level at a specific point upstream of the river power plant, within certain bounds.

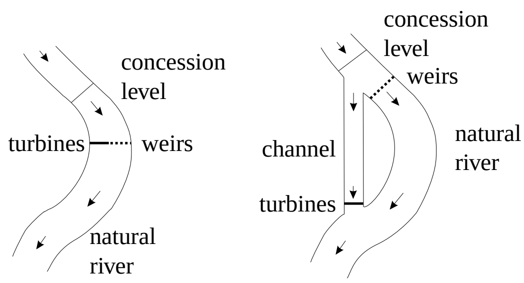

Figure 1: Structures of a basic power plant (left) and a channel power plant (right)

The regulation of the concession level constitutes a control problem with a controller manipulating the discharges through the turbines and the weirs of the power plant. Typically, either the weir or the turbine discharge is kept at a constant value while the discharge through the other facility is adjusted for water level control.

Therefore, from the control point of view, the main power plant components are the turbines, which generate the electrical energy, and the weirs, which drain additional water not flowing through the turbines. In Fig. 1 the structures of a basic river power plant (left) and a channel power plant (right) are given. For a basic river power plant, the turbines and the weirs are located at the same position in the natural river course. For a channel power plant, the turbines are located in a man-made channel in parallel to the natural river course. The concession level positions are also shown, which are usually some tens of meters upstream of the weirs.

The commonly employed control concept consists of a Proportional-Integral (PI) controller with an additional feed-forward term at each power plant (Fig. 2). The difference between the measured value of the concession level h_c and the reference value h_{ref} acts as input to the PI part of the controller. The additional feed-forward term ac-

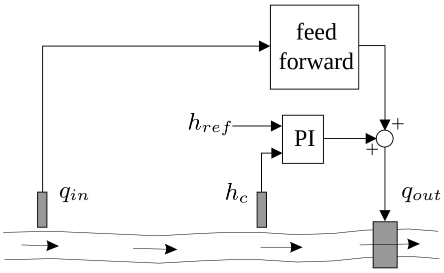

Figure 2: PI controller with an additional feed-forward term

counts for anticipated disturbances by measuring the discharge q_{in} further upstream and applying it with an appropriate delay.

As such controllers are local and no coordination exists between the power plants in a cascade, the consequences of the control actions to downstream power plants are not considered. Often, significant variations are imposed on the discharges in order to keep the concession levels constant. For cascades of power plants, these fluctuations in discharge may be amplified unpredictably above the natural discharge variations in a river. Figure 3 shows an example measured[1] at the river Aare in Switzerland. The considered cascade of six power plants and 50km total length ranges from the town of Murgenthal to the city of Brugg. The inflow to the cascade in Murgenthal shows only small variations, which in the outflow in Brugg are amplified by approximately a factor of five, where large oscillations in both water level and discharge are observed (note the different scales in the two graphs).

These discharge amplifications and the deviations of the concession levels mentioned above are undesirable because they affect nature and may impose problems on navigation. Therefore, besides keeping the concession levels close to their references, the damping of the discharge variations must be considered as an additional objective in the controller design. Since discharge variations are required to control the concession levels, a trade-off between these contradictory control objectives is necessary. With the currently employed controllers this cannot be integrated concisely. The tuning is complicated by the fact that the correct time delay of the feed-forward term is uncertain and the best parameter set of both the PI controller and the feed-forward term depends on the operating point. Additionally, there is no guarantee that the constraints on the concession level are met because they are only implicitly considered by more or less aggressive tuning of the control parameters.

The problem of oscillating discharges in the context of water level control was already discussed in [Neumüller & Bernhauer 1969] and [Neumüller & Bernhauer 1976]. Furthermore, there have been several approaches to the control of cascaded river reaches in the context of irrigation. This is a closely related problem also dealing with open water hydraulics and using water levels and discharges as controlled and manipulated variables. Nevertheless, the control objectives for irrigation are different and mainly aim at a sufficient supply of all drains without wasting water. A review of control and modelling techniques for irrigation systems can be found in [Malaterre & Baume 1998] and shows the variety of proposed approaches. The mostly used approaches apply monovariable PI-type controllers similar to the basic scheme explained above.

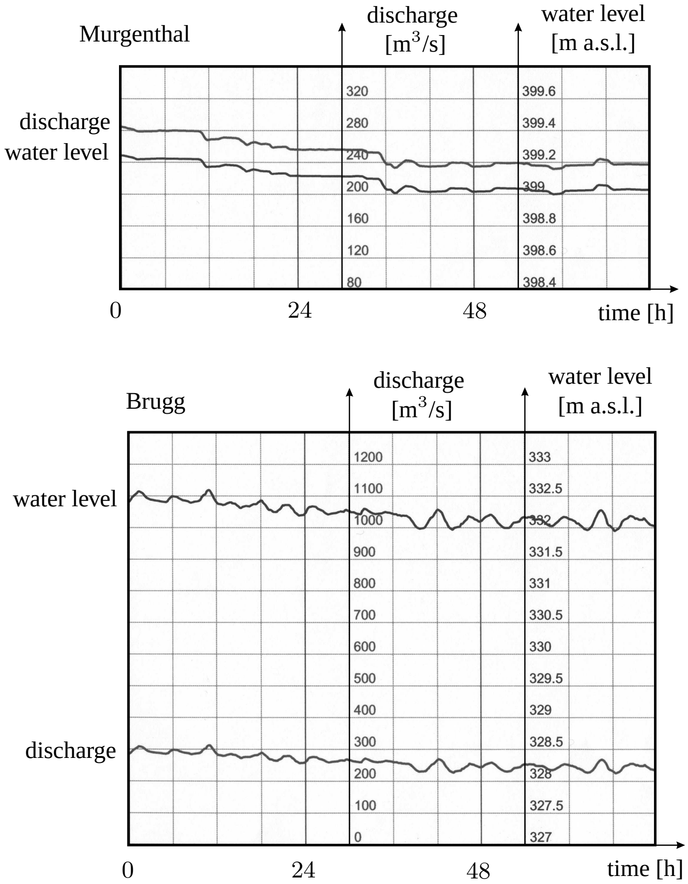

Figure 3: Incoming variations in Murgenthal and amplified variations in Brugg several kilometers downstream

An extensive analysis on PI-controllers with and without additional feed-forward term applied to power plant control is given in [Kühne 1975]. It is shown that the parameters for the PI and the feed-forward part have to be matched carefully in order to achieve a discharge damping and still keep the concession level close to its reference. This tuning is very demanding and even with well matched parameters the damping is only marginal. Similar results are obtained in [Theobald 1999]. There, a simulation program for automated control of cascaded barrages is developed. For that purpose, different control methods have

[1] Measurements provided by the Federal Office for Water and Geology, Switzerland, `http://www.bwg.admin.ch`

been analyzed and implemented. Basically, PI-controllers with feed-forward terms are applied in all of these methods. Supervisory control is not considered.

Multivariable control algorithms including optimal control have been developed in some works [Malaterre & Rodellar 1997], [Malaterre 1998]. However, none of these algorithms deals with constraints, for example the mentioned bounds on water level deviations, and thus they fail to successfully address the considered control problem.

An important component in controller design is the underlying model of the river hydraulics. Also in this field a number of different approaches have been proposed. In most works transfer functions are used to describe the river hydraulics. The approaches range from simple models where rivers are modelled as tanks with delayed inflow [Schuurmans, Bosgra & Brouwer 1995], [Schuurmans, Clemmens, Dijkstra, Hof & Brouwer 1999], [Papageorgiou & Messmer 1989] to sophisticated transfer functions derived from the diffusive wave equation [Litrico & Fromion 2001], [Litrico & Georges 1999]. The advantage of the simpler approaches is a small number of parameters, but this limits on the other hand the modelling capabilities and prevents the modelling of complex effects like damping and reflection of propagating waves. More complex transfer functions with enhanced modelling capabilities usually use a large number of parameters, which lack a direct physical correspondence and are therefore difficult to identify. Hence, extensive experiments in the real cascade are needed for identification.

The contributions of this paper are twofold. One contribution is to derive a generic state space model of an entire cascade of river power plants following the approach in [Chapuis 1998], which is based on the Saint Venant equations. This model accurately captures the river dynamics and the hydraulic coupling between river reaches, while the parameters can be analytically determined from geometric data of the river and from steady state measurements.

A further contribution is the development of a supervisory controller based on Model Predictive Control (MPC) [Maciejowski 2002]. MPC is a control concept, which in process control has become the industrial standard two decades ago. It uses a model of the process to predict the future trajectories of the controlled variables over a horizon in order to determine the optimal sequence of manipulated variables. This is done while explicitly taking constraints on inputs, states and outputs into account. Furthermore, the tuning of the control parameters is straightforward even in the presence of contradictory control objectives. In this paper, MPC is specifically applied to the described control problem. It uses the derived model to predict the future evolution of the water levels and discharges and to determine the discharges through the power plants such that the discharge variations are dampened and the water level constraints are met. To make the proposed control concept applicable to practical power plant arrangements without installing additional costly measurement equipment, the estimation of unmeasurable system states is performed with a standard state estimation scheme. A controller results taking information about all power plants in the cascade into account, coordinating the discharges through the power plants and explicitly considering all constraints. To our best knowledge this approach is new to the control of power plant cascades.

Sect. II illustrates the basic Model Predictive Control concept and Sect. III discusses the general modelling of river hydraulics. Based on this, Sect. IV introduces a generic model of cascades of river power plants and Sect. V discusses the state estimation problem. Section VI shows how MPC can be applied to a power plant cascade. In Sect. VII, simulation results for the developed controller are shown and compared with the current controller leading to the conclusions given in Sect. VIII.

II MODEL PREDICTIVE CONTROL

In the following, we briefly introduce the notion of Model Predictive Control (MPC) for discrete-time linear systems and summarize its basic features. In MPC, the current control input is obtained by solving at each sampling instant an open-loop constrained optimal control problem, using the predictions provided by an internal model of the controlled process. The optimal control problem is formulated over a finite or infinite horizon using the current state of the plant as the initial state. The underlying optimization procedure yields an optimal control sequence that minimizes a given objective function. A *receding horizon* policy is employed, which refers to only applying the first control input of this sequence, and to recomputing the control sequence at the next sampling instant over a shifted horizon, thus providing feedback and closing the control loop. The significant advantages of MPC, including its ability to systematically cope with hard constraints on manipulated variables, states and outputs, and to easily address systems with multiple inputs and outputs, have led to its success and widespread use, which initiated in the process industry more than two decades ago. For details, the reader is referred to [Mayne, Rawlings, Rao & Scokaert 2000] and [Maciejowski 2002].

Consider as model of the controlled process the linear discrete-time system

$$x(k+1) = Ax(k) + Bu(k), \quad (1)$$

$$y(k) = Cx(k), \quad (2)$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^m$ the input vector, $y(k) \in \mathbb{R}^p$ the output vector and $k \in \mathbb{N}$ the discrete time instant. A, B and C are the system, the input and the output matrices respectively, which are of appropriate dimensions. The constraints on the states $x(k)$ and on the inputs $u(k)$ are defined as

$$\underline{x}(k) \leq \quad x(k) \quad \leq \overline{x}(k), \tag{3}$$

$$\underline{u}(k) \leq \quad u(k) \quad \leq \overline{u}(k) \tag{4}$$

with lower limits $\underline{x}(k)$, $\underline{u}(k)$ and upper limits $\overline{x}(k)$, $\overline{u}(k)$.

Next, we define the quadratic cost function

$$\begin{aligned} J(x(0), U_N) \triangleq \sum_{k=0}^{N-1} \left(x^T(k)\mathcal{Q}x(k) + u^T(k)\mathcal{R}u(k)\right) \\ + x^T(N)\mathcal{Q}_t x(N) \end{aligned} \tag{5}$$

as a function of the initial state $x(0)$ and the sequence of control inputs $U_N = [u^T(0), \dots, u^T(N-1)]^T$ over the horizon N. The matrices $\mathcal{Q} \succeq 0$ and $\mathcal{R} \succ 0$ penalize the deviations of the states and inputs from the origin, and $\mathcal{Q}_t$ is referred to as the terminal weight, on which we will elaborate at the end of this section.

The optimal control problem is stated as

$$\min_{U_N} J_N(x(0), U_N) \tag{6}$$

subject to the evolution of the model (1) and (2) with the constraints (3) and (4) for the time steps $k = 0, \dots, N-1$. The optimal control problem (6) is cast as a Quadratic Program (QP), for which very efficient solvers exist. The result is the optimal sequence of control inputs, of which only the first element is implemented.

The above state control problem with finite N and $\mathcal{Q}_t = \mathcal{Q}$ is referred to as the constrained finite time optimal control problem. In general, performance is improved by increasing N, but this also increases the complexity of the underlying QP. Yet, it would be desirable to obtain the solution to the constrained *infinite time* optimal control problem. As briefly summarized in the following, the terminal weight $\mathcal{Q}_t$, a terminal set constraint $x(N) \in \mathcal{X}_{ci}$ and a finite N can be used to emulate the infinite horizon.

Assume that at time step N the state $x(N)$ lies in a set $\mathcal{X}_{ci}$ with the following property. When neglecting the constraints (3) and (4) in the optimal control problem and applying the resulting (unconstrained) control input to the plant, the constraints are met and the state remains in the set for all future time steps, namely $x(k) \in \mathcal{X}_{ci}$ for all $k \geq N$. Such a set is referred to as control invariant set, and the unconstrained optimal control scheme with $N = \infty$ is the so called Linear Quadratic Regulator (LQR). The cost from $k = N$ to infinity for the LQR controller is given by $x^T(N)\mathcal{Q}_t x(N)$, with $\mathcal{Q}_t$ being the

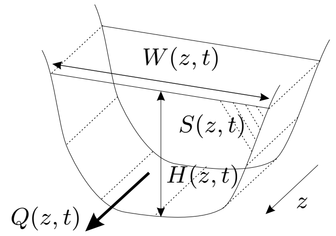

Figure 4: Model parameters for one river cross section

solution to the Discrete Algebraic Riccati equation (ARE)

$$\mathcal{Q}_t = A^T\mathcal{Q}_t A + Q - A^T\mathcal{Q}_t B(B^T\mathcal{Q}_t B + \mathcal{R})^{-1}B^T\mathcal{Q}_t A\,. \tag{7}$$

To ensure $x(N) \in \mathcal{X}_{ci}$ for a finite N, one may either impose $x(N) \in \mathcal{X}_{ci}$ as a constraint, the so called terminal set constraint, or (heuristically) choose N such that it is guaranteed that $x(N) \in \mathcal{X}_{ci}$ holds for all possible $x(0)$.

III MODELLING OF A RIVER REACH

In order to apply MPC to water level control, a model of the cascaded river power plants is required. First, a model of the river hydraulics in a single river reach is derived from the Saint Venant equations following the approach in [Chapuis 1998], which is then used in Sect. IV to build a model of the entire cascade.

By applying the *conservation of volume* and the *conservation of momentum* laws to a river slice, Jean-Claude Saint Venant obtained the partial differential equations

$$0 = \frac{\partial Q}{\partial z} + \frac{\partial S}{\partial t}, \tag{8}$$

$$0 = \frac{1}{g}\frac{\partial}{\partial t}\left(\frac{Q}{S}\right) + \frac{1}{2g}\frac{\partial}{\partial z}\left(\frac{Q^2}{S^2}\right) + \frac{\partial H}{\partial z} + I_f - I_0 \tag{9}$$

which describe the one-dimensional evolution of the water levels and the discharges along a river. Figure 4 illustrates the parameters of these equations in a river cross section. $H(z,t)$ is the water height measured from the river bed, $S(z,t)$ the wetted cross-sectional area, $W(z,t)$ the river top width and $Q(z,t)$ the discharge at position z at time t. The parameter $I_f(z,t)$ is the friction slope accumulating the influence of friction as described in [Chapuis 1998], $I_0(z)$ is the river slope and g is the gravitational constant.

To obtain a linear, discrete time model, the partial differential equations (8) and (9) are linearized and discretized in time and space as elaborated in [Chapuis 1998]. First, the river is approximated by an equivalent rectangular channel. Since the Saint Venant equations are one-dimensional, the exact cross-section geometry is not important as long as the hydraulic properties remain unchanged. A Taylor approximation around the operating

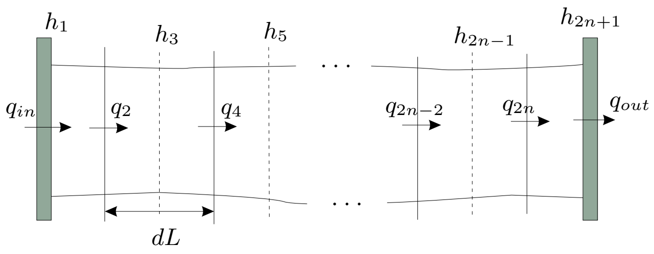

Figure 5: Division of the river into compartments

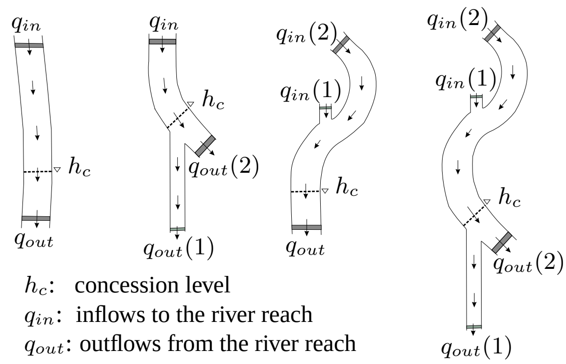

Figure 6: Generic river reaches of which a cascade is composed

point $Q_0(z)$, $H_0(z)$ then leads to a system of linear partial differential equations with the deviations of the water level and of the discharge from their operating point as variables.

For the space discretization the river reach of length L is equally divided into n compartments of length $dL = L/n$ (Fig. 5). The variables q_i and h_i denote the normalized deviations of the water levels and the discharges from the operating point values. The calculation points for these water levels and discharges are shifted by half a compartment length and are placed alternately along the river. The inflow q_{in} and the outflow q_{out} are located at the same positions as h_1 and h_{2n+1}, respectively. Using these calculation points, the partial differential equations are approximated by difference equations with respect to space yielding a linear, discrete space system now consisting of simple differential equations.

To additionally discretize the system in time, zero-order hold time discretization is applied yielding the linear, discrete time state space model

$$\xi(k+1) = \Theta\xi(k) + \Psi u(k), \tag{10}$$
$$\gamma(k) = \Upsilon\xi(k) \tag{11}$$

with

$$\xi(k)=\begin{bmatrix} h_1(k) \\ q_2(k) \\ h_3(k) \\ \vdots \\ q_{2n}(k) \\ h_{2n+1}(k) \end{bmatrix}, \; u(k)=\begin{bmatrix} q_{in}(k) \\ q_{out}(k) \end{bmatrix}, \; \gamma(k)=\begin{bmatrix} h_c(k) \end{bmatrix}, \tag{12}$$

where $\xi(k)$ denotes the state vector, $u(k)$ is the input vector and $\gamma(k)$ is the output vector. The elements in the matrices Θ, Ψ and Υ are calculated from geometrical data of the river and steady state measurements. Thus, no identification experiments are necessary. The dimensions of the vectors in (12) and the matrices Θ, Ψ and Υ depend on the number of discrete sampling points along the considered river reach.

IV CASCADE MODELLING

The system to be modelled is a river of generic geometry containing a cascade of river power plants. The power plants separate different reaches of the river such that the only connection between two successive river reaches are the discharges through the power plant facilities in between. As these discharges are independent of the hydraulic state of the reaches, the river reaches are self-contained systems. Therefore, the derived models of these single river reaches can be combined to a model of an entire cascade. The generic river reaches that are required to compose any combination of power plants with or without man-made channels are given in Fig. 6.

The application of the Saint Venant model described in the previous section to such a river reach is straightforward. For each river reach j the system

$$\xi_j(k+1) = \Theta_j\xi_j(k) + \Psi_j u_j(k), \tag{13}$$
$$\gamma_j(k) = \Upsilon_j\xi_j(k) \tag{14}$$

results, where $\xi_j(k)$, $u_j(k)$ and $\gamma_j(k)$ correspond to the vectors given in (12).

One of the control objectives is to dampen the discharge variations in the river. This is achieved by minimizing the changes in discharge through the power plants. Thus, the model of each river reach is slightly adapted such that the total discharges through the power plants are included in the state vector and the changes of these discharges are used as inputs instead

$$\begin{bmatrix} \xi_j(k+1) \\ u_j(k) \end{bmatrix} = \begin{bmatrix} \Theta_j & \Psi_j \\ 0 & I \end{bmatrix}\begin{bmatrix} \xi_j(k) \\ u_j(k-1) \end{bmatrix} + \begin{bmatrix} \Psi_j \\ I \end{bmatrix} \delta u_j(k), \tag{15}$$
$$\gamma_j(k) = \begin{bmatrix} \Upsilon_j & 0 \end{bmatrix}\begin{bmatrix} \xi_j(k) \\ u_j(k-1) \end{bmatrix}. \tag{16}$$

Like this a penalty can be applied to these changes in the cost function in order to keep them small.

From these river reach models, the model of the entire cascade is built

$$
\begin{aligned}
x(k+1) &= Ax(k) + B\delta u(k), && (17)\\
y(k) &= Cx(k), && (18)
\end{aligned}
$$

where the state, the input and the output vectors for a cascade of m power plants are

$$
x(k)=\begin{bmatrix} \xi_1(k) \\ u_1(k-1) \\ \xi_2(k) \\ u_2(k-1) \\ \vdots \\ \xi_m(k) \\ u_m(k-1) \end{bmatrix}, \delta u(k)=\begin{bmatrix} \delta u_1(k) \\ \delta u_2(k) \\ \vdots \\ \delta u_m(k) \end{bmatrix}, y(k)=\begin{bmatrix} \gamma_1(k) \\ \gamma_2(k) \\ \vdots \\ \gamma_m(k) \end{bmatrix}. \tag{19}
$$

The state space matrices A, B and C are built accordingly from the system matrices of the single river reaches Θ_j, Ψ_j and Υ_j.

V STATE ESTIMATION

As shown in Sect. II, the variables in the state vector have to be updated from measurements after each control step. The elements in the state vector correspond to the water levels and the discharges at the sampling points along the river. Since not all of these values are measurable, they have to be estimated from the available measurements of the concession levels and the headwater levels of the power plants.

The Kalman filter [Kalman 1960] addresses the general problem of estimating the state vector of a discrete time controlled process governed by the linear stochastic difference equation

$$x(k+1) = Ax(k) + B\delta u(k) + \nu(k) \tag{20}$$

with the measurement given by

$$y(k) = Cx(k) + \mu(k)\,. \tag{21}$$

The random variables $\nu(k)$ and $\mu(k)$ represent the process and the measurement noise, respectively. They are assumed to be independent of each other with white and normal (Gaussian) probability distributions. The Kalman filter provides an estimate of the state $\hat{x}(k)$, such that the error covariance matrix

$$P(k) = E\{(x(k) - \hat{x}(k))(x(k) - \hat{x}(k))^T\}\,. \tag{22}$$

is minimized. Often, the measurement noise can be obtained experimentally. The covariances of the noises can be used to reflect the dominant source of uncertainty. Specifically, they express the trade off between the credibility of the obtained measurements with respect to the predictions resulting from the stochastic model dynamics (20). A further elaboration on the concept of the Kalman filter is beyond the scope of this paper and the reader is referred to one of the numerous control theory textbooks.

The estimation of the non-measurable variables allows for a controller implementation which only uses available measurements such as the discharges through weirs and turbines, and the water levels on their upstream side. No installation of additional costly measurement equipment is necessary.

VI MPC FOR CASCADE CONTROL

Using the derived model of a power plant cascade, a constrained optimal control problem as described in Sect. II can be formulated for the control of a cascade. Such a supervisory control scheme allows for a coordinated optimization of the control inputs of all power plants.

The control objectives are to keep the concession levels as close to the reference as possible and to dampen the variations in discharge through the power plants facilities. As these demands are contradictory, a trade-off between both criteria is necessary. Moreover, the authorities impose limits within which the concession levels may vary. These limits may be violated only for a short time or under extraordinary circumstances like floods or heavy rainfalls and are therefore defined as soft constraints. Concerning the turbines and weirs, physical limitations on the minimal and maximal discharge and on their maximal rate of change exist. These represent hard constraints, which cannot be violated.

For the modelling of these constraints the concession levels

$$x_c(k) = G_c x(k) = \begin{bmatrix} h_{c1}(k) & \dots & h_{cm}(k) \end{bmatrix}^T \tag{23}$$

and the manipulated discharges

$$x_u(k) = G_u x(k) = \begin{bmatrix} u_1(k) & \dots & u_m(k) \end{bmatrix}^T \tag{24}$$

are considered. Since they are contained in the state vector $x(k)$, the constraints on these variables are state constraints. The constraints on the concession levels $x_c(k)$ are modelled as soft constraints with slack variables $\varepsilon_c(k)$ and are formulated as

$$
\begin{aligned}
\underline{x}_c(k) - \varepsilon_c(k) \leq \quad & x_c(k) \quad \leq \overline{x}_c(k) + \varepsilon_c(k), && (25)\\
0 \leq \quad & \varepsilon_c(k). && (26)
\end{aligned}
$$

The hard constraints on the manipulated discharges $x_u(k)$ and the changes in the manipulated discharges $\delta u(k)$ are given by

$$
\begin{aligned}
\underline{x}_u(k) \leq \quad x_u(k) \quad &\leq \overline{x}_u(k), && (27) \\
\underline{\delta u}(k) \leq \quad \delta u(k) \quad &\leq \overline{\delta u}(k). && (28)
\end{aligned}
$$

The constrained optimal control problem amounts to

$$
\begin{aligned}
\min_{\delta U} \sum_{k=0}^{N-1} \big(x^T(k)\mathcal{Q}x(k) + \delta u^T(k)\mathcal{R}\delta u(k) + \\
+ \varepsilon_c^T(k)\mathcal{Q}_\varepsilon \varepsilon_c(k) \big) + x^T(N)\mathcal{Q}_t x(N) \qquad (29)
\end{aligned}
$$

subject to the model (17), (18) and the constraints (25)-(28).

The penalty matrix $\mathcal{Q}$ has non-zero elements only on the diagonal in the positions corresponding to the deviations of the concession levels. The matrices $\mathcal{R}$ and $\mathcal{Q}_\varepsilon$ are both diagonal and of dimension equal to the number of power plants in the cascade. The minimization of the first term accounts for keeping the deviations of the concession levels from their reference values small. At the same time, the changes in the power plant discharges are penalized in the second term in order to dampen the discharge variations. The tuning of the controller is reduced to assigning values to $\mathcal{Q}$, $\mathcal{R}$ and $\mathcal{Q}_\varepsilon$ expressing the relative importance of the contradictory control objectives. Finally, $\mathcal{Q}_t$ solves the Discrete Algebraic Riccati equation and N is chosen large enough to ensure that (29) resembles the infinite horizon control problem.

Since the resulting cost function is quadratic, the optimal control sequence can be determined from (29) by solving a standard Quadratic Program (QP). Various efficient software packages for this purpose are available.

VII SIMULATION RESULTS

For performance evaluation, the developed control concept is applied as an example to a cascade in the river Aare. The topology of the river is shown in Fig. 7. The river hydraulics are simulated with the state-of-the-art simulation software FLORIS[2] based on detailed river data. This data consists of 1684 cross section profiles and steady state measurements of water levels and discharges at the respective sampling points.

The MPC tuning parameters are shown in Table 1. As opposed to the very detailed river model in FLORIS, which serves as the controlled process, the river reach models used for the MPC scheme require a significantly lower amount of river data. A model with a total number of 209 states and the five turbine discharges as inputs is used in

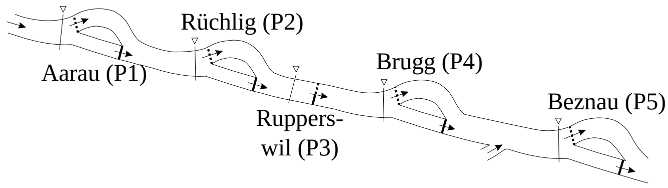

Figure 7: Power plant cascade at the river Aare

this simulation. The number of model states can be further reduced to 150 using a standard model reduction technique [Moore 1981]. A standard Kalman filter is used to estimate the states of the model from the available measurements. The sampling interval is 72s and the MPC horizon is set to 50 time steps (=1h). This approximately corresponds to the propagation delay from the upstream to the downstream end of the cascade. The computation time to solve the QP in each step on a standard PC is roughly 8.6s, which is very short compared to the available length of the sampling interval.

Parameters	**Values**
number of states	35/35/41/45/53
sampling interval	72s
N	50 time steps
$\underline{x}_c$, $\overline{x}_c$	$\pm$2cm for all plants
$\mathcal{R}$	diag([0.01 0.01 0.01 0.1 1])
$\mathcal{Q}$	diag([0.002 ... 0.002])
$\mathcal{Q}_\varepsilon$	diag([10 ... 10])

Table 1: MPC tuning parameters for the five considered power plants

The control parameters are tuned with focus on discharge damping of the cascade as a whole, which corresponds to the damping of the discharge at the last power plant. Therefore, the largest weight of 1 is assigned to changes in the discharge of the last power plant (P5). As the discharge changes at the other power plants are of minor interest, the respective penalty for the fourth power plant is ten times lower (0.1) and the changes in the discharge of the first three power plants are penalized with even smaller weights (0.01). To fully utilize the available storage volume for disturbance damping, the weight on the concession level deviations is very small (0.002), which merely ensures that the concession level is eventually driven back to zero and does not remain at the limits in steady state. The slack variables are heavily penalized with 10 in order to not violate the imposed concession level bounds during regular operation. Note that none of these parameters depends on the operating point or the expected disturbances, thus leading to a straightforward tuning.

The currently employed PI-type controllers are applied for comparison. Their parameters are tuned by our indus-

[2]Scietec, developer and distributor of the river simulation software FLORIS, http://www.scietec.com

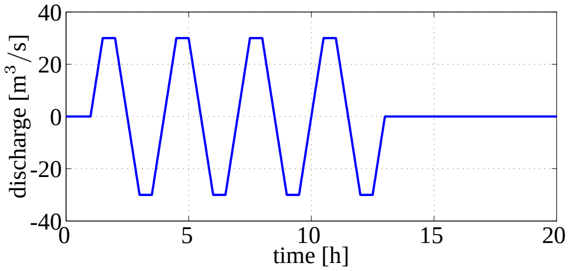

Figure 8: Inflow disturbance to the cascade

trial partner[3] using their commonly employed procedure based on inflow and outflow experiments at the different stages of the cascade.

To point out the advantages of MPC over local PI controllers, one example simulation is dicussed here in detail. For this simulation, the inflow to the cascade is chosen to vary around a steady state discharge of $200\mathrm{m}^3/\mathrm{s}$. The applied disturbance is approximately sinusoidally shaped with an amplitude of $30\mathrm{m}^3/\mathrm{s}$ (Fig. 8). Such discharge variations appear frequently in the considered power plant cascade.

Around the operating point of $200\mathrm{m}^3/\mathrm{s}$, the hydraulic situation is very sensitive to incoming disturbances and difficult to be handled by the current controllers. The turbines of all power plants are not fully utilized and are thus in charge of the concession level control. The weir discharges are held constant during the entire simulation.

Figure 9 shows the resulting deviations of the concession levels and the turbine discharges from the operating point at the five power plants (P1-P5) for the incoming disturbance shown in Fig. 8. The results for the local PI controllers are shown as dash-dotted lines, the results for the supervisory MPC controller as solid lines. The horizontal lines in the left figures indicate the maximum allowed concession level deviations of ± 2cm.

For the PI-type controllers, the power plants in the cascade are not coordinated. Thus, each power plant can only react on disturbances that have already reached the measuring point of its feed-forward term. This drastically limits the possiblities for anticipating control. The main control objective of keeping the concession level constant or bringing it quickly back to its reference value in case of deviations is achieved at the expense of poor discharge damping, or even discharge amplification. As a consequence, the discharge variations are amplified as they propagate through the cascade. To achieve a damping of the discharge variations, the PI controllers would have to be tuned less aggressively (e.g. for plant P3). Such a tuning might yield concession levels varying exactly within the prescribed limits. Yet, this would only hold for the specific disturbance and for the specific operating point.

[3]Rittmeyer AG, Zug, http://www.rittmeyer.ch

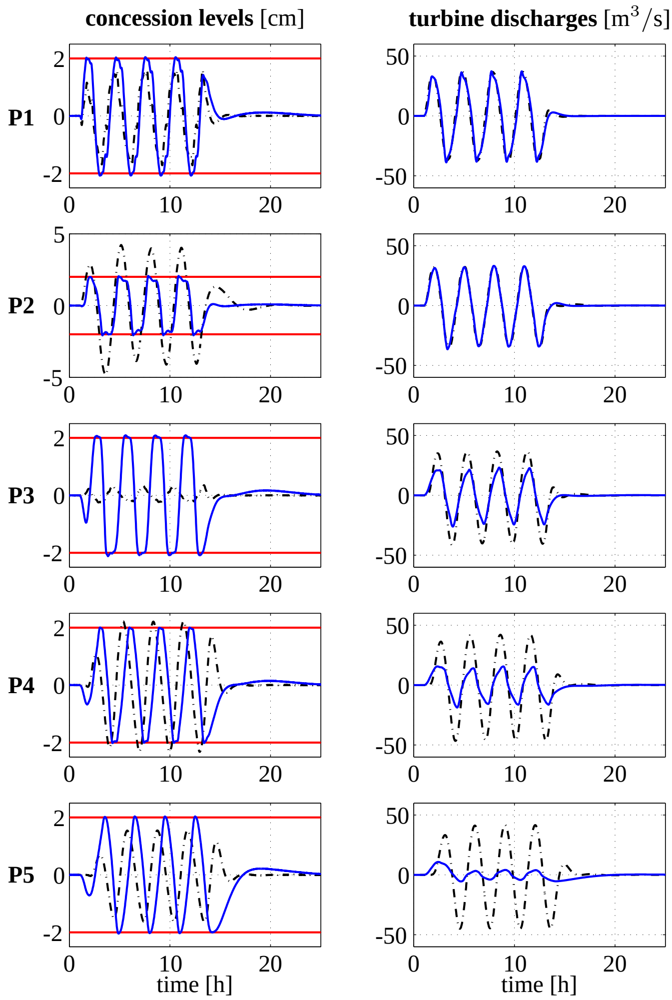

Figure 9: Concession levels and turbine discharges of plants P1-P5 for PI controllers (dash-dotted) and MPC controller (solid)

When varying the disturbance or the operating point, this tuning would be inappropriate, leading to either constraint violations or unsatisfactory discharge damping.

As opposed to the PI-type controllers, MPC coordinates its control actions for all power plants. Already at time instant $t = 1$h, the beginning of the disturbance, the discharges through the turbines at all power plants are increased. The consequence is that the concession levels are lowered for later compensation of the propagating disturbance. Especially at the power plants P3-P5, this is clearly visible in Fig. 9. The anticipating and coordinating feature of the controller allows for damping the discharges from one power plant to the next one, such that at the fifth power plant P5 the initial discharge variations of $\pm 30\mathrm{m}^3/\mathrm{s}$ are reduced to about $\pm 5\mathrm{m}^3/\mathrm{s}$. Because of the comparably small storage volume in the second river reach (between P1 and P2), the second power plant P2 achieves only a marginal damping. Nevertheless, it keeps its concession level within the specified bounds, which are severely violated when using the PI controller. Since MPC explicitly takes the level constraints into account, it fully utilizes the allowed deviations of the concession levels in order to achieve the maximum possible discharge damping. Overall, the control performance is enhanced significantly compared to the PI-type controllers.

VIII CONCLUSIONS

This paper presented a supervisory controller for water level control of cascaded river power plants. A Model Predictive Control (MPC) scheme was proposed solving a constrained optimal control problem, which was formulated based on a river model derived from the Saint Venant equations.

Closed-loop simulations with a numerically simulated river as controlled process showed that the supervisory MPC controller significantly improves the damping of discharge variations compared to the current control scheme. The anticipating control actions are coordinated for the entire cascade and the interactions between the power plants are taken into consideration. Moreover, the contradictory control objectives and all constraints are taken into account, which is not possible with the currently employed PI controllers that were designed to control water levels without considering discharge damping. The tuning of the proposed controller is straightforward and can easily be adapted to special hydraulic situations or emergency cases. This results in a robust and reliable water level control concept for cascades of river power plants.

Since the proposed hydraulics model is generic and allows for the integration of additional in- and outflows, it is applicable to various practical situations including cascades of storage power plants and irrigation systems. The flexibility in formulating the control problem allows for integrating additional objectives and constraints such as economical criteria. Financial objectives considering the amount of produced power at a certain time are particularly interesting in cases where the operational and environmental constraints are less restrictive.

A practical implementation of the proposed concept is feasible using standard computer hardware and process control systems as already installed at most power plants. In the considered cascade, all power plants are equipped by the same company and communication lines for a supervisory controller are already available, which makes the solution particularly interesting.

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