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Model Predictive Control in Power Electronics: A Hybrid Systems Approach

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Abstract— **The field of power electronics poses challenging control problems that cannot be treated in a complete manner using traditional modelling and controller design approaches. The main difficulty arises from the hybrid nature of these systems due to the presence of semiconductor switches that induce different modes of operation and operate with a high switching frequency. Since the control techniques traditionally employed in industry feature a significant potential for improving the performance and the controller design, the field of power electronics invites the application of advanced hybrid systems methodologies. The computational power available today and the recent theoretical advances in the control of hybrid systems allow one to tackle these problems in a novel way that improves the performance of the system, and is systematic and implementable. In this paper, this is illustrated by two examples, namely the Direct Torque Control of three-phase induction motors and the optimal control of switch-mode dc-dc converters.**

I. INTRODUCTION

Power electronics systems represent a well-established technology that has seen significant performance improvements over the last two decades. In general, these systems are used to transform electrical power from one – usually unregulated – form to another regulated one (consider e.g. the problem of unregulated dc to regulated dc conversion). This transformation is achieved by the use of semiconductor devices that operate as power switches, turning on and off with a high switching frequency. From the control point of view, power electronics circuits and systems constitute excellent examples of hybrid systems, since the discrete switch positions are associated with different continuous-time dynamics. Moreover, physical and safety constraints are present.

Traditionally, power electronics circuits and systems have been controlled in industry using linear controllers combined with non-linear procedures like Pulse Width Modulation (PWM). The models used for controller design are a result of simplifications that include averaging the behavior of the system over time (to avoid modelling the switching) and linearizing around a specific operating point disregarding all constraints. As a result, the derived controller usually performs well only in a neighborhood around the operating point. To make the system operate in a reliable way for the whole operating range, the control circuit is subsequently augmented by a number of heuristic patches. This procedure requires large development times and lacks theoretically backed guarantees for the operation of the system.

Nowadays, however, the recent theoretical advances in the field of hybrid systems, and the significant increase in computational power available for the control of power electronics systems are inviting both the control and the power electronics communities to revisit the control issues associated with power electronics applications. Motivated by this, a novel control methodology for power electronics systems is outlined in this paper, where we demonstrate the application of hybrid constrained optimal control methodologies to power electronics systems. More specifically, we show how Model Predictive Control (MPC) [1] can be applied to induction motor drives and dc-dc conversion, using two examples as illustration: the Direct Torque Control (DTC) of three-phase induction motors and the optimal control of fixed-frequency switch-mode dc-dc converters.

The use of constrained optimal control methodologies implies the solution of an underlying optimization problem. Given the high switching frequency used in power electronics applications and the large computation times that are usually needed to solve such problems, may very well make an on-line solution approach infeasible. Depending on the application, this obstacle can be overcome in two ways; either by pre-solving off-line the optimization problem for the whole state-space using multi-parametric programming – a procedure that results in a polyhedral PieceWise Affine (PWA) controller that can be stored in a look-up table, or by developing solution algorithms that are dedicated and tailored to the problem and can thus be executed within the limited time available. The first approach has been followed here for the constrained optimal control problem of fixed-frequency dc-dc converters, whereas the second one has been applied to the DTC problem.

The paper is organized as follows: Section II gives an overview of the theoretical framework used, including the basic ideas behind the off-line solution of the optimal control problem. Subsequently, we present the new modelling and optimal control approaches to the DTC problem in Section III and to the control problem of dc-dc converters in Section IV. Conclusions and an outlook are provided in Section V.

II. OPTIMAL CONTROL OF HYBRID SYSTEMS

In the following, we restrict ourselves to the discrete-time domain, and we confine our models to (piecewise) affine dynamics rather than allowing general nonlinear dynamics. This not only avoids a number of mathematical problems (like Zeno behavior), but allows us to derive models for which we can pose analysis and optimal control problems

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that are computationally tractable. To model such discrete-time linear hybrid systems, we adopt Mixed Logical Dynamical (MLD) [2] models and the PieceWise Affine (PWA) [3] framework. Other representations of such systems include Linear Complementarity (LC) systems, Extended Linear Complementarity (ELC) systems and Max-Min-Plus-Scaling (MMPS) systems that are, as shown in [4], equivalent to the MLD and PWA forms under mild assumptions.

Model Predictive Control (MPC) [1] has been used successfully for a long time in the process industry and recently also for hybrid systems, for which, as shown in [2], MPC has proven to be particularly well suited. The control action is obtained by minimizing an objective function over a finite or infinite horizon subject to the evolution in time of the model of the controlled process and constraints on the states and manipulated variables. For linear hybrid systems, depending on the norm used in the objective function, this minimization problem amounts to solving a *Mixed-Integer Linear Program* (MILP) or *Mixed-Integer Quadratic Program* (MIQP).

The major advantage of MPC is its straightforward design procedure. Given a (linear or hybrid) model of the system, only an objective function incorporating the control objectives needs to be set up. Additional hard (physical) constraints can be easily dealt with by adding them as inequality constraints, whereas soft constraints can be accounted for in the objective function using large penalties. Moreover, MPC adapts well to different physical setups and allows for a unified approach. For details concerning the set up of the MPC formulation in connection with linear hybrid models, the reader is referred to [2] and [5]. Details about MPC can be found in [1].

To make the proposed constrained optimal control strategies applicable to power electronics systems it is mandatory to overcome the obstacle posed by the large computation times occurring when solving the optimal control problem on-line. This can be achieved by pre-computing the optimal state-feedback control law off-line for all feasible states using the state vector as a parameter. For hybrid systems, such a method has been recently introduced, which is based on a PWA description of the controlled system and a linear objective function, using the 1- or ∞-norm. The details can be found in [6], where the authors report an algorithm that generates the solution by combining dynamic programming with multi-parametric programming and some basic polyhedral manipulations. As shown in [7], the resulting optimal state-feedback control law is a PWA function of the state defined on a polyhedral partition of the feasible state-space. More specifically, the state-space is partitioned into polyhedral sets and for each of these sets the optimal control law is given as an affine function of the state. As a result, such a state-feedback controller can be easily implemented on-line as a look-up table.

III. OPTIMAL DIRECT TORQUE CONTROL OF THREE-PHASE INDUCTION MOTORS

The rapid development of power semiconductor devices led to the increased use of adjustable speed induction motor drives in a variety of applications. In these systems, dc-ac inverters are used to drive induction motors as variable frequency three-phase voltage or current sources. One methodology for controlling the torque and speed of induction motor drives is Direct Torque Control (DTC) [8], which features very favorable control performance and implementation properties.

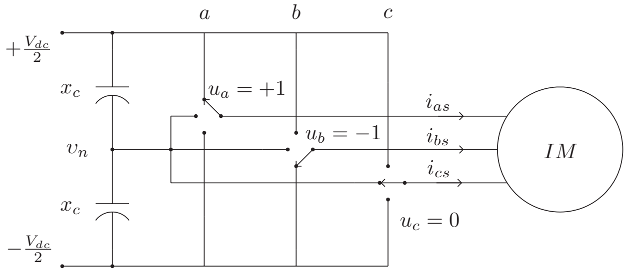


Fig. 1. The equivalent representation of a three-phase three-level inverter driving an induction motor (IM), where V_{dc} is the voltage of the dc-link, v_n is the neutral point potential, and u_a, u_b, $u_c \in \{-1, 0, 1\}$ are the switch positions of the phases a, b and c, respectively

The basic principle of DTC is to exploit the fast dynamics of the motor's stator flux and to directly manipulate the stator flux vector such that the desired torque is produced. This is achieved by choosing an inverter switch combination that drives the stator flux vector to the desired position by directly applying the appropriate voltages to the motor windings. This choice is usually made every $T_s = 25\,\mu$s using a pre-designed switching table. Traditionally, the latter is derived in a heuristic way and – depending on the particularities of the application – addresses a number of different control objectives. These primarily concern the induction motor – more specifically, the stator flux and the electromagnetic torque need to be kept within pre-specified bounds around their references. In high power applications, where three-level inverters with Gate Turn-Off (GTO) thyristors are used, the control objectives are extended to the inverter and also include the minimization of the average switching frequency and the balancing of the inverter's neutral point potential around zero. As mentioned in the introduction, because of the discrete switch positions of the inverter, the DTC problem is intrinsically a hybrid control problem, which is complicated by the nonlinear behavior of the torque, length of the stator flux and the neutral point potential.

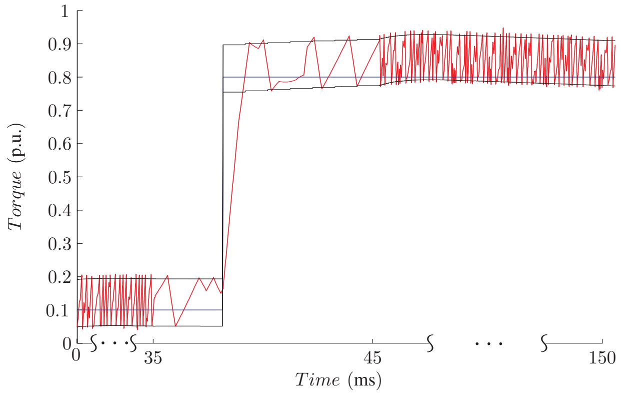
We aim at deriving MPC schemes that keep the three controlled variables (torque, flux, neutral point potential) within their given bounds, minimize the (average) switching frequency, and are conceptually and computationally simple yet yield a significant performance improvement with respect to the state of the art. More specifically, the term *conceptually simple* refers to controllers allowing the straightforward tuning of the controller parameters or even a lack of such parameters, and easy adaptation to different physical setups and drives, whereas *computationally simple* implies that the control scheme does not require excessive computational power to allow the implementation on DTC hardware that is currently available or at least will be so within a few years.

An important first step is to derive discrete-time hybrid models of the drive tailored to our needs – or more specifically, models that are of low complexity yet of sufficient accuracy to serve as prediction models for our model-based control schemes. To achieve this, we have exploited in [9], [10] a number of physical properties of DTC drives. These properties are the (compared with the stator flux) slow rotor flux and speed dynamics, the symmetry of the voltage vectors, and the invariance of the motor outputs under flux rotation. The low-complexity models are derived by assuming constant speed within the prediction horizon, mapping the states (the fluxes) into a 60 degree sector, and aligning the rotor flux vector with the d-axis of the orthogonal dq0 reference frame rotating with the rotational speed of the rotor [11]. The benefits of doing this are a reduction of the number of states from five to three, and a highly reduced domain on which the nonlinear functions need to be approximated by PWA functions.

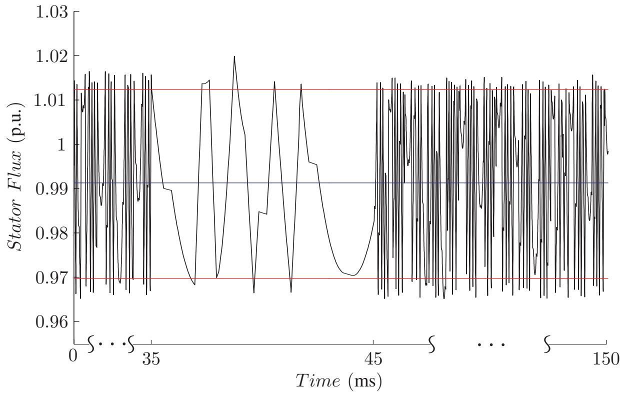
Based on the hybrid models of the DTC drive, we have proposed in [10], [12], [13] three novel control approaches to tackle the DTC problem, which are inspired by the principles of MPC and tailored to the peculiarities of DTC. For comparing with the industrial state of the art, we have used for all our simulations the Matlab/Simulink model of ABB's ACS6000 DTC drive [14] containing a squirrel-cage rotor induction motor with a rated apparent power of 2 MVA and a 4.3 kV three-level dc-link inverter. This model was provided to us by ABB in the framework of our collaboration and its use ensures a realistic set-up.

A. *DTC based on Priority Levels*

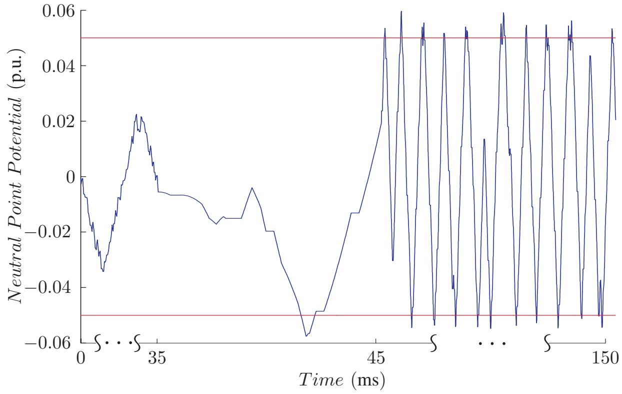
The first scheme [10] uses soft constraints to model the hysteresis bounds on the torque, stator flux and neutral point potential, and approximates the average switching frequency (over an infinite horizon) by the number of switch transitions over a short horizon. To make this approximation meaningful and to avoid excessive switching, one needs to enforce that switch transitions are only performed if absolutely necessary, i.e. when refraining from switching would lead to a violation of the bounds on the controlled variables within one time-step. This means that the controller has to postpone any scheduled switch transition until absolutely necessary. This strategy can be implemented by imposing a time-decaying penalty on the switch transitions, where switch transitions within the first time-step of the prediction interval result in larger penalties then those that are far in the future. Moreover, three penalty levels with corresponding penalties of different orders of magnitude provide clear controller priorities and make the fine-tuning of the objective function obsolete. To extend the prediction interval without increasing the computational burden, we propose to use a rather long prediction interval, but a short prediction horizon. This is achieved by finely sampling the prediction model with T_s only for the first steps, but more coarsely with a multiple of T_s for steps far in the future. This approach is similar to utilizing the technique of blocking control moves and leads to a time-varying prediction model with different sampling rates.



(a) Electromagnetic torque



(b) Stator flux



(c) Neutral point potential

Fig. 2. Closed-loop simulation of the DTC scheme based on priority levels during a step change in the torque reference. Note the different scaling of the time-axis

Simulation results demonstrating the behavior of the controlled variables are presented in Fig. 2. This control scheme not only leads to short commissioning times for DTC drives, but it also leads to a performance improvement in terms of a reduction of the switching frequency in the range of 20 % with respect to the industrial state of the art, while simultaneously reducing the torque and flux ripples. Yet the complexity of the control law is rather excessive [9].

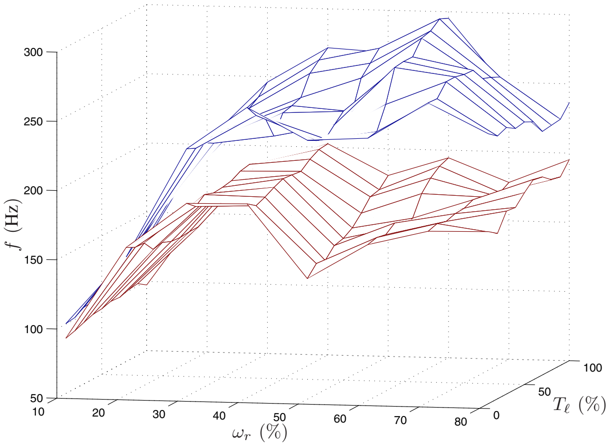


Fig. 3. Comparison of switching frequency f of ABB's DTC (upper surface) with respect to MPC based on extrapolation (lower surface) over the grid of operating points, given by the rotational speed ω_r and the load torque T_ℓ

B. DTC based on Feasibility and Move Blocking

The second scheme, presented in [12], exploits the fact that the control objectives only weakly relate to optimality but rather to feasibility, in the sense that the main objective is to find a control input sequence that keeps the controlled variables within their bounds, i.e. a control input sequence that is feasible. The second, weaker objective is to select among the set of feasible control input sequences the one that minimizes the average switching frequency, which is again approximated by the number of switch transitions over the (short) horizon. We therefore propose an MPC scheme based on feasibility in combination with a move blocking strategy, where we allow for switching only at the current time-step. For each input sequence, we determine the number of steps the controlled variables are kept within their bounds, i.e. remain feasible. The switching frequency is emulated by the cost function, which is defined as the number of switch transitions divided by the number of predicted time-steps an input remains feasible, and the control input is chosen so that it minimizes this cost function.

As shown in [12], the simplicity of the control methodology translates into a state-feedback control law with a complexity that is of an order of magnitude lower than the one of the first scheme, while the performance is improved.

C. DTC based on Extrapolation

The third scheme [13] can be interpreted as a combination of the two preceding concepts. Specifically, we use a rather short horizon and compute for the input sequences over the horizon the evolution of the controlled variables using the prediction model. To emulate a long horizon, the "promising" trajectories are extrapolated and the number of steps is determined when the first controlled variable hits a bound. The cost of each input sequence is then determined by dividing the total number of switch transitions in the sequence by the length of the extrapolated trajectory. Minimizing this cost yields the optimal input sequence and the next control input to be applied.

The major benefits of this scheme are its superior performance in terms of switching frequency, which is reduced over the whole range of operating points by up to 50 %, with an average reduction of 25 %. This performance improvement is shown in Fig. 3, where the switching frequency of the developed control scheme is compared with the one achieved with ABB's currently employed approach [14]. Furthermore, the controller needs no tuning parameters.

Summing up, at every discrete sampling instant, all control schemes use an internal model of the DTC drive to predict the output response to input sequences, choose the input sequence that minimizes an approximation of the average switching frequency, and apply only the first element of the input sequence according to the receding horizon policy thus providing feedback. Moreover, the proposed schemes are tailored to a varying degree to the specific DTC problem set-up. Starting from the first scheme, the complexity of the controllers in terms of computation time and memory requirement for the controller hardware was reduced by several orders of magnitude, while the performance was steadily improved. Since the switching losses of the inverter are roughly proportional to the switching frequency, the performance improvement in terms of the switching frequency reduction translates into energy savings and thus into a more cost efficient operation of the drive, which is especially important because high power applications are considered here. Most importantly, the last control scheme (based on extrapolation) is currently being implemented by our industrial partner ABB who has also protected this scheme by a patent application [13].

IV. OPTIMAL CONTROL OF DC-DC CONVERTERS

Switch-mode dc-dc converters are switched circuits that transfer power from a dc input to a load. They are used in a large variety of applications due to their light weight, compact size, high efficiency and reliability. Since the dc voltage at the input is unregulated (consider for example a coarsely rectified ac voltage) and the output power demand changes significantly over time (resulting in a time-varying load), the control objective is to achieve output voltage regulation in the presence of input voltage and output load variations.

Fixed-frequency switch-mode dc-dc converters use semiconductor switches that are periodically switched on and off, followed by a low-pass filtering stage with an inductor and a capacitor to produce at the output a dc voltage with a small ripple. Specifically, the switching stage comprises a primary semiconductor switch that is always controlled, and a secondary switch that is operated dually to the primary one. The switches are driven by a pulse sequence of constant frequency (period), the *switching frequency* f_s (*switching period* T_s), which characterizes the operation of the converter. The dc component of the output voltage can be regulated through the duty cycle d, which is defined by $d = \frac{t_{on}}{T_s}$, where t_{on} represents the interval within the switching period during which the primary switch is in conduction. Therefore,

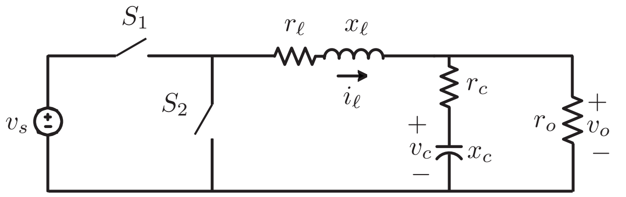


Fig. 4. Topology of the step-down synchronous converter, where v_s is the unregulated input voltage, S_1 and S_2 are the dually operated switches, i_ℓ is the inductor current, v_c is the capacitor voltage, and v_o is the regulated output voltage over the output resistance r_o

the main control objective for dc-dc converters is to drive the primary switch with a duty cycle such that the dc component of the output voltage is equal to its reference. For details the reader is referred to the standard power electronics literature (e.g. [15]).

The difficulties in controlling dc-dc converters arise from their hybrid nature. In general, these converters feature three different modes of operation, where each mode is associated with a (different) linear continuous-time dynamic law. Furthermore, constraints are present resulting from the converter topology. In particular, the manipulated variable (duty cycle) is bounded between zero and one, and in the discontinuous current mode a state (inductor current) is constrained to be non-negative. Additional constraints are imposed as safety measures, such as current limiting or soft-starting, where the latter constitutes a constraint on the maximal derivative of the current during start-up. The control problem is further complicated by gross changes in the operating point due to input voltage and output load variations, and model uncertainties.

Motivated by the hybrid nature of dc-dc converters, we have presented in [16] and [17] a novel approach to the modelling and controller design problem for fixed-frequency dc-dc converters, using a synchronous step-down dc-dc converter as an illustrative example (see Fig. 4). In particular, the notion of the ν-resolution model was introduced to capture the hybrid nature of the converter, which led to a PWA model that is valid for the whole operating regime and captures the evolution of the state variables within the switching period.

Based on the converter's hybrid model, we formulated and solved an MPC problem. The control objective were to regulate the output voltage to its reference, and to minimize changes in the duty cycle (to avoid limit cycles at steady state) while respecting the safety constraint (on the inductor current) and the physical constraint on the duty cycle (which is bounded by zero and one). This allows for a systematic controller design that achieves the objective of regulating the output voltage to the reference despite input voltage and output load variations while satisfying the constraints. In particular, the control performance does not degrade when changing the operating point.

To make the scheme implementable, we derived offline the explicit PWA state-feedback control law with 121 polyhedra. This controller can be easily stored in a look-up table and used for the practical implementation of the proposed control scheme. The derived control law, for the set of converter and controller parameters considered in [17], is shown in Fig. 5, where one can observe the control input $d(k)$ as a PWA function of the two transformed states i'_ℓ (inductor current) and v'_o (output voltage).

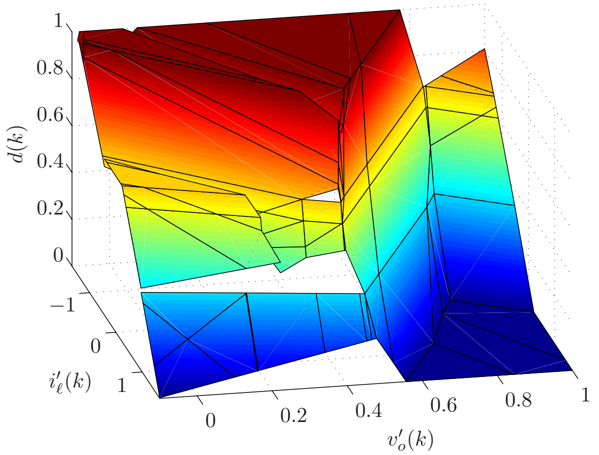


Fig. 5. State-feedback control law: the duty cycle $d(k)$ is given as a PWA function of the transformed state vector; dark blue corresponds to $d(k) = 0$ and dark red to $d(k) = 1$

The transformed states correspond to a scaling of the actual measured states over the input voltage. This allows us to account for changes in the input voltage that are an important aspect of the control problem. Moreover, the output load may change drastically (basically in the whole range from open- to short-circuit). This is addressed by adding an additional parameter to the control problem formulation and a Kalman filter is used to adjust it. For more details on these considerations and the reasoning behind the use of the output voltage (rather than the capacitor voltage) as a state in the prediction model, the reader is referred to [18].

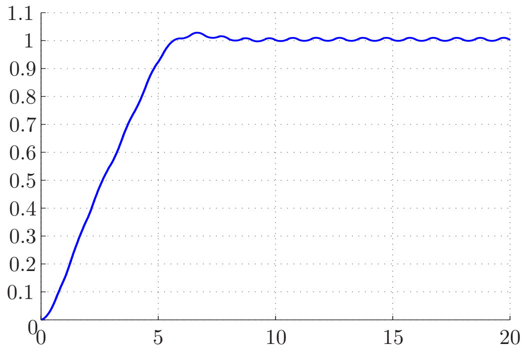
Regarding the performance of the closed-loop system, the simulation results in Fig. 6 show the step response of the converter in nominal operation during start-up. The output voltage reaches its steady state within 10 switching periods with an overshoot that does not exceed 3%. The current limit is basically respected by the peaks of the inductor current. The small deviations observed are due to the approximation error introduced by the coarse resolution of the ν-resolution model (here $\nu = 3$ subperiods). The same holds for the small – in the range of 0.5% – steady-state error that is present in the output voltage.

Moreover, an a posteriori analysis shows that the considered state-space is a positively invariant set under the derived optimal state-feedback controller. Most importantly, a PieceWise Quadratic (PWQ) Lyapunov function can be computed that proves exponential stability of the closed-loop system for the whole range of operating points.

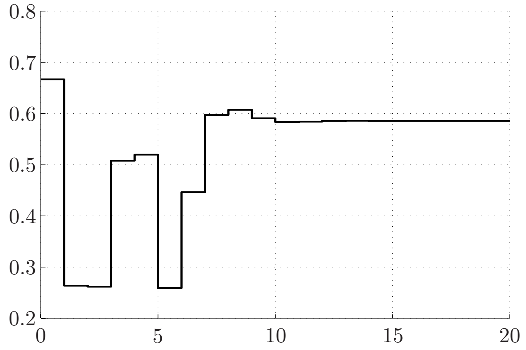
V. CONCLUSIONS AND OUTLOOK

In this paper, we have outlined a number of new control approaches for power electronics circuits and systems that

(a) Inductor current $i_\ell(t)$



(b) Output voltage $v_o(t)$



(c) Duty cycle $d(t)$

Fig. 6. Closed-loop response during start-up in nominal operation

are based on hybrid systems and constrained optimal control methodologies. Two cases have been considered, namely the Direct Torque Control of three-phase induction motors and the optimal control of fixed-frequency dc-dc converters.

Our intention is to show that hybrid system methods can be successfully applied to industrially relevant power electronics control problems, bringing benefits in terms of system design and performance. On the other hand, the major issue that arises is the complexity of the developed control algorithms. It is the opinion of the authors, however, that tailoring the control methods to the specific problem set-up leads to controllers with high performance and modest complexity. This fact, in combination with the continuous increase in the computational power available for controlling these systems, enables the control and power electronics communities to revisit some of the traditionally established methods in a more theoretically rigorous and systematic way.

VI. ACKNOWLEDGEMENTS

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