

# A Model for a Novel Variable Speed Cage Induction Generator

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**Abstract-** This paper presents a novel cage induction generator and a mathematical model, through which its behavior can be accurately predicted. The novelty of the generator is due mainly to the constant frequency electricity generation, without an intermediate inverter stage, by a 3-phase cage induction machine operated at varying rotor speeds. The technique uses any one of the three stator phases of the machine as the excitation winding, and the remaining two phases, connected in series, as the power winding. The two series-connected and one isolated (TSCAOI) phase winding configuration magnetically decouples the two sets of windings, enabling independent control. Electricity is generated through the power winding with appropriate excitation to the isolated single winding at any desired frequency, using a simple single-phase square-wave inverter or a reversible rectifier, when the rotor is driven at variable speed. The mathematical model, which accurately represents the proposed generator, is implemented in Matlab/Simulink. Simulations of the TSCAOI power generation under various operating conditions suggest that the proposed technique is viable, and is expected to gain popularity being simple, low-in-cost and not requiring output filtering for grid-integration.

## I. INTRODUCTION

Use of renewable energy and its economic viability have not been the focus in the past because of the availability of low cost fossil energy, which was in abundance. However excessive, unnecessary and inefficient use of fossil energy has since become a global concern, owing to rapidly decreasing fossil resources, rising fuel prices, increasing demand for energy, and more importantly, the awareness of global warming and environmental impact. Consequently, it has now become a common practice of governing bodies to place more emphasis on energy saving, harnessing renewable energy, when and where possible, and energy management through efficient generation, conversion, transmission and distribution. This initiative incited a new area of active research and development within both academia and industry under the context of 'Green or Clean' energy.

Many techniques, through which mechanical to electrical energy conversion can be realized, have been proposed and developed with commercial success. These techniques vary from one to the other with different levels of sophistication, characteristics, performance, cost etc, and are suitable for range of energy sources and applications from very low to very high power levels. Amongst the various renewable energy sources that are available, wind energy can be

considered as a source that has been widely used. Wind turbine systems have been in use for power levels ranging from megawatt level down to mini or even microwatt level, where miniature sensors are powered. In these systems, various techniques have been employed within the system for mechanical to electrical energy conversion. Fig. 1 shows several electricity generation schemes, based on induction generators, employed in typical wind turbine systems.

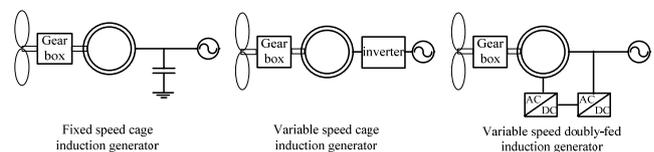


Fig. 1 Typical induction generators used in wind turbines

Of the schemes illustrated in Fig. 1, fixed speed cage induction generators are well known for their simplicity and low cost, and operated at constant rotor speed to generate power at constant frequency for both direct grid integration and stand alone operation, [1-7]. Usually they are excited (or rotor magnetization is provided) through a capacitor when operated at constant speed, and are incapable of tracking maximum power of a turbine at various wind speeds with varying rotor speeds unless an intermediate inverter stage is employed to allow for the variable speed operation. Such an additional inverter stage, rated essentially to the same power level of the generator itself, is often found to be not economically justifiable for some applications.

According to literature, an electricity generation scheme, based on a variable speed cage induction machine without an intermediate inverter stage, is yet to be reported. This paper presents a novel technique, whereby a 3-phase cage induction machine can be used as a generator under variable speed conditions without an intermediate inverter stage. The technique uses one of the three windings in isolation for excitation and the remaining two, connected in series, as the power winding for single phase electricity generation. Alternatively, the two series connected windings may also be used for excitation while the power is generated through the isolated single winding. A 3-phase cage induction machine is modeled mathematically in the proposed two series connected and one isolated (TSCAOI) phase winding configuration, and

simulation results indicate that the machine can be operated both at sub-synchronous and super-synchronous rotor speeds to generate electricity at constant frequency. The proposed technique allows for energy storage through the excitation winding, and is expected to gain popularity, particularly in small scale applications, being relatively simple and low in cost.

## II. PROPOSED NOVEL CAGE INDUCTION GENERATOR

Cage induction machines are undoubtedly the work horse of industry, and can still be regarded as a serious competitor to permanent magnet machines, being self-starting, rugged, reliable, efficient, and maintenance free to offer a long trouble-free working life. Of these cage induction machines, three-phase machines are significantly less expensive, more efficient and smaller in frame size in comparison to their single-phase counterpart of similar power ratings. Consequently, 3-phase cage induction motors are economically more appealing and have thus become the preferred choice for numerous applications, even at de-rated power levels as encountered in Steinmetz configuration [8-9].

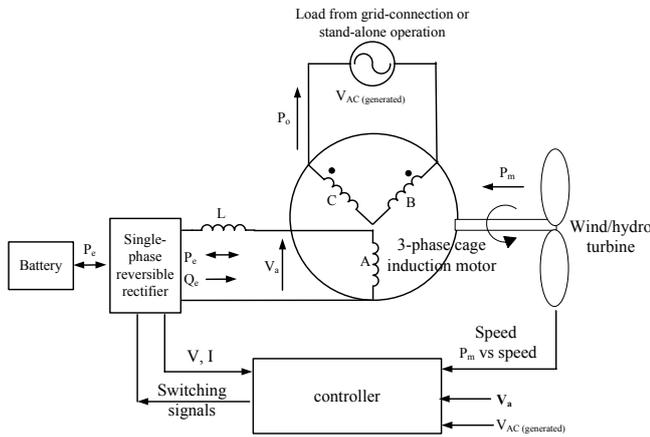


Fig. 2 Proposed TSCAOI winding configuration

The novel technique [10-11] proposed in this paper also uses a 3-phase cage induction machine, exploiting its economical advantage, to generate single-phase electricity at variable rotor speeds without an intermediate inverter stage. The technique configures the three stator windings of the 3-phase cage induction machine in a novel way to create a separate excitation winding and a power winding. In this configuration, any one of the three phase windings is used solely in isolation for excitation while the remaining two are connected in series to generate power at a desired frequency while the rotor is driven at any given speed. Alternatively, the machine can also be configured in such a way that the two series connected windings provide the excitation while the single winding generates. The proposed two series-connected and one isolated (TSCAOI) winding configuration of a 3-phase cage induction machine is shown in Fig. 2. As shown

mathematically in the following section, TSCAOI winding configuration magnetically decouples both excitation and power windings from each other, and thus allows for independent control as in the case of a single-phase induction motor with an auxiliary winding [6].

In the proposed technique, excitation for the generator is provided through the single winding using either a square-wave inverter or a reversible rectifier, connected to a battery. The former is the simplest, and can be operated at the desired generation frequency using a less sophisticated controller to provide the required reactive power of the generator. In the latter case, as shown in Fig. 2, the system is more sophisticated and complex but facilitates bi-directional power flow, allowing for both energy storage and later retrieval. The level of excitation in both cases is determined by the voltage generated in the power winding. A controller, comprising a voltage feed back with phase-lock-loop (PLL), can be employed to regulate the excitation. The controller in the simplest form may provide only the reactive power requirement of the generator (not the load), and at more sophisticated level may be used to control both the active and reactive power flow in accordance with the phase angle and voltage magnitude between the inverter and the excitation winding.

## III. MATHEMATICAL MODEL

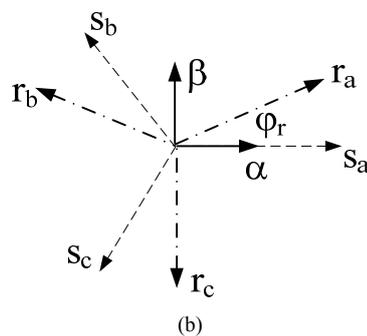
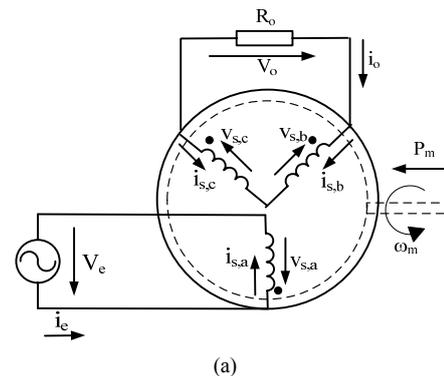


Fig. 3 (a) TSCAOI model (b) stator and rotor with respect to the ' $\alpha\beta$ ' frame

Fig. 3(a) shows a 3-phase cage induction machine configured in the proposed TSCAOI winding arrangement

with no closed loop control. For model derivation, it is assumed that 'α' axis of 'αβ' frame is aligned with phase 'a' of the stator windings as shown in Fig. 3(b). If the rotor phase 'a' is assumed to be at an angle,  $\varphi_r$ , from the α axis, rotor quantities can then be transformed into 'αβ' frame using transformation,

$$[K_r] = \frac{2}{3} \begin{bmatrix} \cos(\varphi_r) & \cos(\varphi_r + \frac{2\pi}{3}) & \cos(\varphi_r - \frac{2\pi}{3}) \\ \sin(\varphi_r) & \sin(\varphi_r + \frac{2\pi}{3}) & \sin(\varphi_r - \frac{2\pi}{3}) \end{bmatrix} \quad (1)$$

According to the TSCAOI configuration, the relationship between the voltages and currents in the power and excitation windings and those in the stator phase windings can be given by

$$[v_{s,eo}] = [Q][v_{s,abc}] \quad (2)$$

$$[i_{s,eo}] = [Q][i_{s,abc}] \quad (3)$$

Where

$$[v_{s,eo}] = \begin{bmatrix} v_{se} \\ v_{so} \end{bmatrix}, \quad [i_{s,eo}] = \begin{bmatrix} i_{se} \\ i_{so} \end{bmatrix}, \quad [v_{s,abc}] = \begin{bmatrix} v_{sa} \\ v_{sb} \\ v_{sc} \end{bmatrix},$$

$$[i_{s,abc}] = \begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{bmatrix}, \quad [Q] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}, \quad [Q]^{-1} = [Q]^T$$

A 3-phase cage induction machine can be represented in the 'abc' frame by the following standard equations,

$$[v_{s,abc}] = [R_s][i_{s,abc}] + p\{[L_s][i_{s,abc}]\} + p\{[L_{sr}][i_{r,abc}]\} \quad (4)$$

$$[v_{r,abc}] = [R_r][i_{r,abc}] + p\{[L_{sr}]^T[i_{s,abc}]\} + p\{[L_r][i_{r,abc}]\} \quad (5)$$

Where 'p' is the differential operator,  $[v_{r,abc}]$  and  $[i_{r,abc}]$  are defined according to  $[v_{s,abc}]$  and  $[i_{s,abc}]$ ,  $[v_{r,abc}] = 0$  for cage machines, and

$$[R_s] = \begin{bmatrix} r_s & 0 & 0 \\ 0 & r_s & 0 \\ 0 & 0 & r_s \end{bmatrix}, \quad [R_r] = \begin{bmatrix} r_r & 0 & 0 \\ 0 & r_r & 0 \\ 0 & 0 & r_r \end{bmatrix}$$

$$[L_s] = \begin{bmatrix} (L_{ls} + L_{ms}) & -L_{ms}/2 & -L_{ms}/2 \\ -L_{ms}/2 & (L_{ls} + L_{ms}) & -L_{ms}/2 \\ -L_{ms}/2 & -L_{ms}/2 & (L_{ls} + L_{ms}) \end{bmatrix}$$

$$[L_r] = \begin{bmatrix} (L_{lr} + L_{mr}) & -L_{mr}/2 & -L_{mr}/2 \\ -L_{mr}/2 & (L_{lr} + L_{mr}) & -L_{mr}/2 \\ -L_{mr}/2 & -L_{mr}/2 & (L_{lr} + L_{mr}) \end{bmatrix}$$

$$[L_{sr}] = L_{ms} \begin{bmatrix} \cos(\varphi_r) & \cos(\varphi_r + \frac{2\pi}{3}) & \cos(\varphi_r - \frac{2\pi}{3}) \\ \cos(\varphi_r - \frac{2\pi}{3}) & \cos(\varphi_r) & \cos(\varphi_r + \frac{2\pi}{3}) \\ \cos(\varphi_r + \frac{2\pi}{3}) & \cos(\varphi_r - \frac{2\pi}{3}) & \cos(\varphi_r) \end{bmatrix}$$

In these equations, parameters  $r_s$ ,  $r_r$ ,  $L_{ls}$ ,  $L_{ms}$ ,  $L_{lr}$ ,  $L_{mr}$  and  $L_{sr}$  are the stator resistance, rotor resistance, stator leakage, stator magnetization, rotor leakage, rotor magnetization and stator-to-rotor mutual inductance, respectively, referred to the stator side. The stator and rotor parameters in the 'abc' frame can now be transformed into the 'eo' and 'αβ' frames, respectively.

$$[v_{s,eo}] = [Q][R_s][Q]^{-1}[i_{s,eo}] + [Q]p\{[L_s][Q]^{-1}[i_{s,eo}]\} + [Q]p\{[L_{sr}][K_r]^{-1}[i_{r,\alpha\beta}]\} \quad (6)$$

And

$$[v_{r,\alpha\beta}] = [K_r][R_r][K_r]^{-1}[i_{r,\alpha\beta}] + [K_r]p\{[L_{sr}]^T[Q]^{-1}[i_{s,eo}]\} + [K_r]p\{[L_r][K_r]^{-1}[i_{r,\alpha\beta}]\} \quad (7)$$

After lengthy manipulations with appropriate substitutions, (6) and (7) can be re-written in the following form

$$[v_{s,eo}] = r_s \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} [i_{s,eo}] + \begin{bmatrix} L_{ls} + L_{ms} & 0 \\ 0 & 2L_{ls} + 3L_{ms} \end{bmatrix} p[i_{s,eo}] + \frac{3}{2}L_{ms} \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{3} \end{bmatrix} p[i_{r,\alpha\beta}] \quad (8)$$

$$0 = L_{ms}\omega_r \begin{bmatrix} 0 & \sqrt{3} \\ -1 & 0 \end{bmatrix} [i_{s,eo}] + L_{ms} \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{3} \end{bmatrix} p[i_{s,eo}] \quad (9)$$

$$r_r[i_{r,\alpha\beta}] + (L_{lr} + \frac{3}{2}L_{ms})\omega_r \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} [i_{r,\alpha\beta}] + (L_{lr} + \frac{3}{2}L_{ms})p[i_{r,\alpha\beta}]$$

Where  $\omega_r$  is the electrical rotor speed in rad/s. To complete the machine model, it is necessary to select state variables and derive the appropriate equations for integration. In this case, the elements of the machine current vector are chosen as the state variables.

Equation (10) shows the state space model using the winding currents as the phase vector, as derived from (8) and (9)

$$p[i] = [A][i] + [B][v] \quad (10)$$

Where

$$[i] = [i_{se} \quad i_{so} \quad i_{r\alpha} \quad i_{r\beta}]^T,$$

$$[v] = [v_{s,eo}] = [v_{se} \quad v_{so}]^T$$

$$[A] = \begin{bmatrix} -\frac{r_s L_{tr}}{D_1} & \frac{2L_M^2 \omega_t}{\sqrt{3}D_1} & \frac{L_M r_r}{D_1} & \frac{L_M L_{tr} \omega_t}{D_1} \\ -\frac{L_M^2 \omega_t}{\sqrt{3}D} & -\frac{r_s L_{tr}}{D} & -\frac{\sqrt{3}L_M L_{tr} \omega_t}{2D} & \frac{\sqrt{3}L_M r_r}{2D} \\ \frac{2r_s L_M}{3D} & -\frac{2L_M(L_{ls} + \frac{2}{3}L_M)\omega_t}{\sqrt{3}D} & -\frac{r_r(L_{ls} + \frac{2}{3}L_M)}{D} & -\frac{L_{tr}(L_{ls} + \frac{2}{3}L_M)\omega_t}{D} \\ \frac{2L_M L_{ss} \omega_t}{3D} & \frac{2r_s L_M}{\sqrt{3}D} & \frac{L_{ss} L_{tr} \omega_t}{D} & -\frac{L_{ss} r_r}{D} \end{bmatrix}$$

$$[B] = \begin{bmatrix} \frac{L_{tr}}{D_1} & 0 \\ 0 & \frac{L_{tr}}{2D} \\ -\frac{2L_M}{3D} & 0 \\ 0 & -\frac{L_M}{\sqrt{3}D} \end{bmatrix}$$

$$L_M = \frac{3}{2}L_{ms}, \quad L_{ss} = L_{ls} + L_M, \quad L_{tr} = L_{lr} + L_M$$

$$D = (L_{ss}L_{tr} - L_M^2)$$

$$D_1 = L_{ls}L_{tr} + \frac{2}{3}L_{lr}L_M$$

The electromagnetic torque of the machine can be derived from

$$T_e = \frac{P}{2} [i_{s,abc}] \frac{\partial}{\partial \phi_r} \{ [L_{sr}] [i_{r,abc}] \} \quad (11)$$

where P denotes the number of poles. Equation (11) in 'abc' quantities is transformed into the 'eo' and ' $\alpha\beta$ ' frame, and can be given by

$$T_e = \frac{P}{2} L_M (\sqrt{3} i_{so} i_{r\alpha} - i_{se} i_{r\beta}) \quad (12)$$

Equation (12) represents the torque components due to both load and excitation currents. At steady state, the torque given in (12) is equal to the turbine torque. The equation of motion of the generator is given by

$$p\omega_r = \frac{P}{2} \frac{1}{J} (T_p - T_e) \quad (13)$$

Where J (kgm<sup>2</sup>) is the inertia and T<sub>p</sub> (Nm) is the torque of the prime mover.

#### IV. MATLAB/SIMULINK MODEL

Behavior of the machine in the proposed TSCAOI generator configuration is investigated simulating the above mathematical model using Matlab/Simulink interface. Fig. 4 shows the Simulink block of the proposed generator. Only open loop operation of the generator is first considered with a resistive load for a given excitation and turbine torque.

Simulated results of close loop control are expected to be presented in the final version of the paper. A 3kW prototype generator system is currently being developed and experimental results are also expected to be available to include in the final paper.

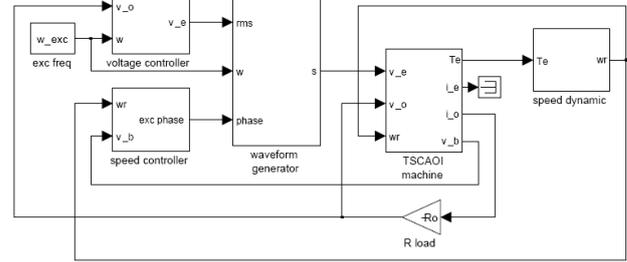
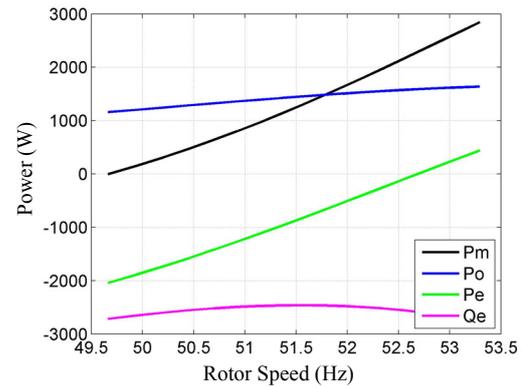


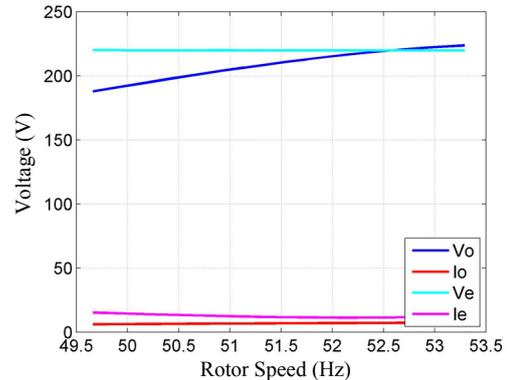
Fig. 4 Simulink block diagram

#### V. RESULTS

A 4-pole, 3kW and 400V cage induction machine, configured in the proposed TSCAOI arrangement, was simulated in open loop for various operating conditions. The parameters of the machine are given in the Appendix.

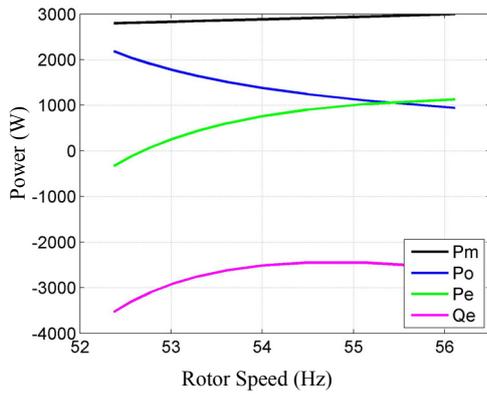


(a) power vs rotor speed

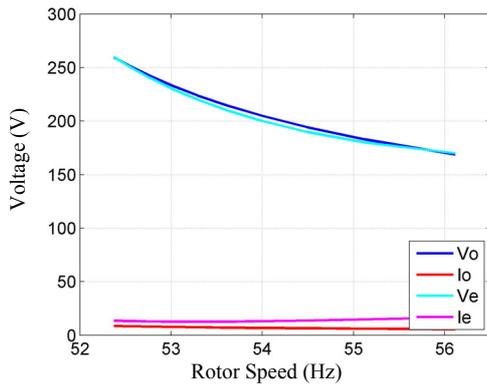


(b) voltage vs rotor speed

Fig. 5 power and voltage for different rotor speeds (for T<sub>p</sub> = 1-17 Nm, V<sub>e</sub> = 220 V and R<sub>o</sub> = 30 Ohms)



(a) power vs rotor speed



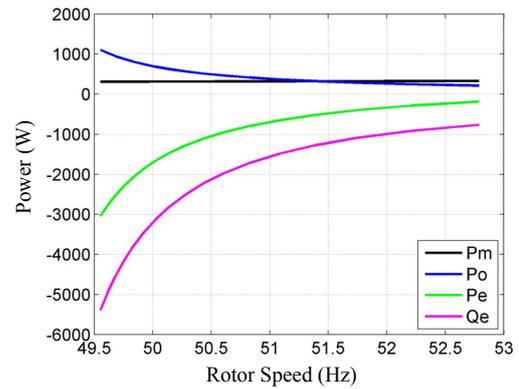
(b) voltage vs rotor speed

Fig. 6 power and voltage for different rotor speeds (for  $T_p = 17 \text{ Nm}$ ,  $V_e = 260\text{-}170 \text{ V}$  and  $R_o = 30 \text{ Ohms}$ )

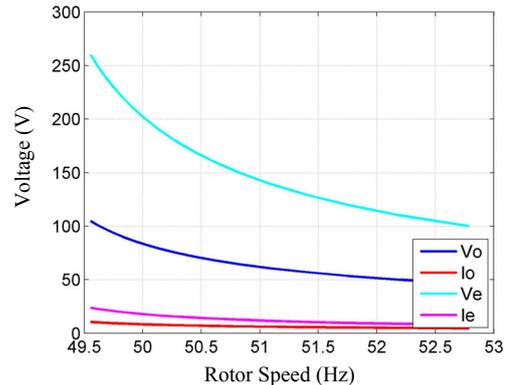
Fig. 5(a) shows the power-frequency relationship of the generator when the prime mover torque was varied. A single winding was used for the 200V/50Hz excitation while the two series connected produced power at 50Hz. As evident, the output power is supplied by both the prime mover and the excitation source until the rotor electrical frequency exceeds 52.5 Hz. As the rotor speeds up with more prime mover power, the power generated does not increase as the output voltage cannot further increase under the fixed excitation being unable to meet the var requirement. Consequently, the excitation winding absorbs power. The voltage and current profile for different rotor speeds under this situation are shown in Fig. 5(b).

Fig. 6 shows the performance of the generator for a high and constant turbine torque. In this situation the excitation provided was decreased, and as a result the same magnitude of output voltage cannot be maintained and the output power decreases. The excess power is thus absorbed by the excitation winding.

Performance of the generator under low and constant turbine torque is illustrated in Fig. 7. As the input turbine power is low, the output power required at high excitation, or consequently at high output voltage, is largely provided by the excitation source.



(a) power vs rotor speed



(b) voltage vs rotor speed

Fig. 7 power and voltage for different rotor speeds (for  $T_p = 2 \text{ Nm}$ ,  $V_e = 100\text{-}260 \text{ V}$  and  $R_o = 10 \text{ Ohms}$ )

As the excitation drops, the power supplied by the excitation, too, drops in accordance with the decreasing output voltage.

Fig. 8 and Fig. 9 illustrate the situation when the excitation is provided by the two series connected winding and the power is generated through the single winding. In contrast to the situation in Fig. 6, where the excitation was provided through the single winding, the generated output voltage is appeared to be low. Hence the power output of the generator is less than that produced with single winding excitation.

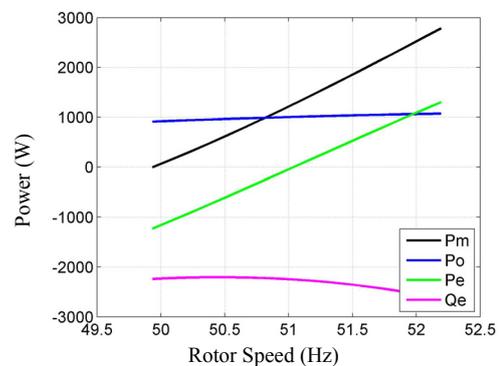


Fig. 8 power for different rotor speeds (for  $T_p = 0\text{-}17 \text{ Nm}$ ,  $V_e = 400 \text{ V}$  and  $R_o = 30 \text{ Ohms}$ )

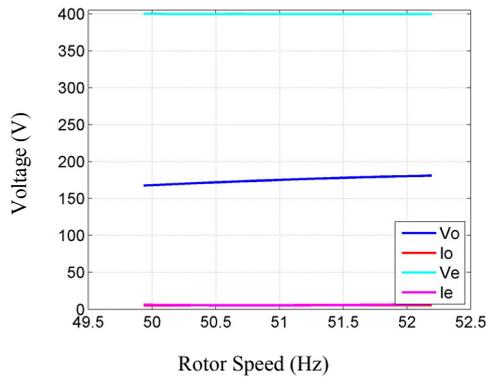


Fig. 9 Voltage for different rotor speeds (for  $T_p = 0-17$  Nm,  $V_e = 400$  V and  $R_o = 30$  Ohms)

## VI. CONCLUSION

A novel cage induction generator and a mathematical model, which predicts its behavior, have been presented. The validity of the proposed concept of generation has been verified using simulations. Results indicate that the technique is viable, and allows for the generation of electricity at constant frequency while the cage induction machine is operated at variable speed. Although the proposed generator appears to be capable of operating both at sub-synchronous and super-synchronous rotor speeds, and absorbing or supplying real power through the excitation winding, a closed-loop control strategy would be more appropriate to identify its limitations. Currently, closed-loop control, through which the output voltage and rotor speed can be regulated, is being implemented and these results are expected to be presented in the final version together with measured results of a 3kW prototype unit, which is also under construction at present.

## APPENDIX

Machine : cage type 3 kW, 4-pole 400V/50Hz induction motor, which has the following parameters.

$$\begin{aligned}
 R_s &= 1.5 \Omega \\
 R_r &= 2 \Omega \\
 L_{ls} &= 0.011 \text{ H} \\
 L_{lr} &= 0.011 \text{ H} \\
 L_M &= 0.214 \text{ H} \\
 J &= 0.01 \text{ kgm}^2
 \end{aligned}$$

## ACKNOWLEDGMENT

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