Fast Computation of Optimized Pulse Patterns for Multilevel Converters

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***Abstract*—This paper presents a method for the fast calculation of multilevel optimized pulse patterns (OPPs) with relaxed symmetry properties. To achieve this, the OPP optimization problem is reformulated and a single optimization problem is solved instead of multiple ones. As a result, the computation time is kept modest, making it possible to compute OPPs with relaxed symmetry and thus leverage their benefits.**

***Index Terms*—Optimal modulation, pulse width modulation (PWM), multilevel converters**

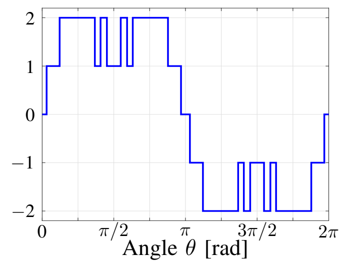
I. Introduction

OPTIMIZED pulse patterns (OPPs) produce load currents of the highest possible quality [1]. This is done by computing the optimal pulse pattern (i.e., switching angles and switching transitions) in an offline optimization procedure that aims to minimize the load current distortions.

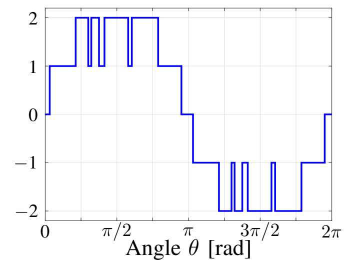
Typically, OPPs are characterized by quarter- and half-wave symmetry (QaHWS). As shown in [2] for three-level converters, relaxing the symmetry to half-wave symmetry (HWS) increases the search space, allowing for improved harmonic distortions. This observation was extended to five-level converters in [3]. However, unlike three-level OPPs, which have a unique switching sequence, there are several switching sequences available for OPPs with more levels (five and above). As a result, the optimal multilevel OPP is found by solving the associated optimization problem for every possible switching sequence, with the optimal sequence being determined in a post-processing step [4]. Given this, and considering that relaxing the quarter-wave symmetry significantly increases the number of possible switching sequences, it can be understood that the HWS multilevel OPP problem can require an exorbitant amount of computation time.

To tackle the high computation times of multilevel OPPs, [5] proposed a reformulation of the optimization problem by incorporating both the OPP switching angles and switching transitions in one optimization variable, called virtual angle. Alas, the proposed virtual angles require QaHWS, rendering them unsuitable for the computation of HWS OPPs.

In this work, an alternative formulation of virtual angles is proposed inspired by the method presented in [6] for selective harmonic elimination. As shown with the presented numerical results, the proposed method enables the fast computation of multilevel HWS OPPs as it speeds up the optimization process by more than 90% compared to the conventional method.



(a) QaHWS OPP



(b) HWS OPP

Fig. 1: Examples of five-level OPPs with different symmetry properties for $d = 5$ at modulation index $m = 1.05$.

II. OPPs with Relaxed Symmetry

The single-phase switched waveform $u(\theta)$ of conventional κ-level OPPs, with $\kappa = 3, 5, 7, \ldots$, has QaHWS. As a result, it suffices to know the d switching angles $\alpha_i \in [0, \frac{\pi}{2}]$, $i \in \{1, \ldots, d\}$, along with the d switching transitions $\Delta u_i = u_i - u_{i-1} \in \{-1, 1\}$ to fully describe $u(\theta)$, where u_i is the i^{th} switch position, and d is the pulse number. On the other hand, when HWS OPPs are of concern, $2d$ switching angles $\alpha_i \in [0, \pi]$, $i \in \{1, \ldots, 2d\}$, and $2d$ switching transitions Δu_i are needed to specify $u(\theta)$. Note that $u_0 = 0$ holds for both QaHWS and HWS OPPs. An example of five-level OPPs with different symmetry properties is depicted in Fig. 1.

The optimal switching patterns are calculated by minimizing an objective function that captures the current total demand distortion (TDD). Assuming an inductive load, the latter is proportional to the weighted sum of the pulse pattern harmonics $\hat{u}_n = \sqrt{a_n^2 + b_n^2}$. The analytical expressions of the Fourier coefficients a_n and b_n can be found in [2]. It is worth noting that even harmonics are zero due to the HWS.

The conventional solution method is to enumerate all possible switching sequences for a given d, and subsequently solve the following problem for each one of them

$$\begin{aligned} \underset{\boldsymbol{\alpha}}{\text{minimize}} \quad & J(\boldsymbol{\alpha}) = \textstyle\sum_{n=5,7,\ldots} \frac{a_n^2 + b_n^2}{n^2} \\ \text{subject to} \quad & a_1 = 0\,, \quad b_1 = \tfrac{\kappa-1}{2} m \\ & 0 \leq \alpha_1 \leq \alpha_2 \leq \ldots \leq \alpha_D \leq A\,, \end{aligned} \tag{1}$$

where $\boldsymbol{\alpha} = [\alpha_1 \; \alpha_2 \; \ldots \; \alpha_d]^T$, $D = d$ and $A = \frac{\pi}{2}$ for QaHWS, while $\boldsymbol{\alpha} = [\alpha_1 \; \alpha_2 \; \ldots \; \alpha_{2d}]^T$, $D = 2d$ and $A = \pi$ for HWS. Note that $m \in [0, 4/\pi]$ is the desired modulation index. Only non-triplen, odd harmonics are considered in (1)

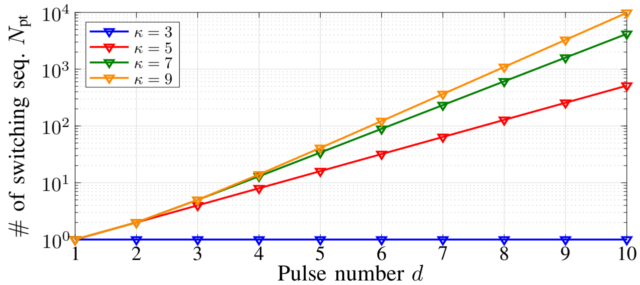


Fig. 2: Number of possible κ-level HWS switching sequences.

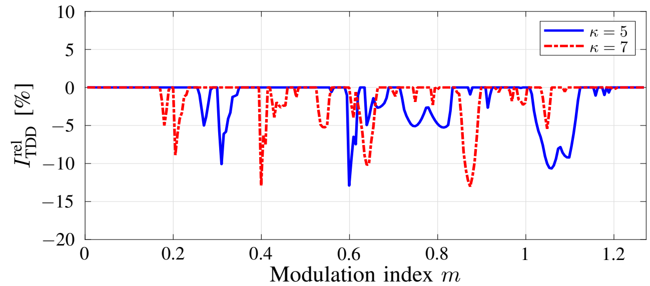


Fig. 3: Relative current TDD of five-level OPPs (solid blue line) and seven-level OPPs (dash-dotted red line) with $d = 7$.

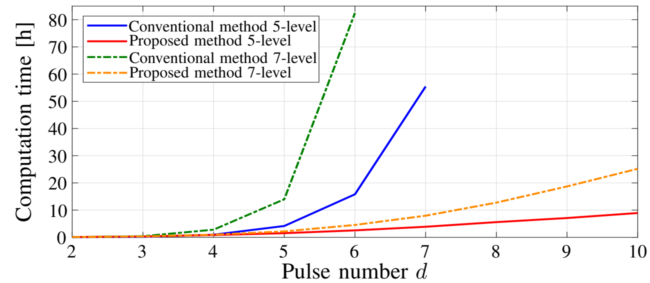


Fig. 4: Computation times of five- (solid lines) and seven-level (dash-dotted lines) HWS OPPs with the conventional and proposed methods.

as triplen harmonics do not drive harmonic current when a wye-connected load with a floating star-point is assumed.

This approach is extremely time-consuming when $\kappa \geq 5$ because the number N_{pt} of the switching sequences that need to be considered, and consequently the computation time, increase exponentially with the pulse number d, see Fig. 2. Therefore, the computation of HWS OPPs becomes practically impossible with the conventional solution method.

To allow for the computation of HWS OPPs, this work solves *only one* optimization problem by introducing a set of new optimization variables called virtual angles $\gamma_i \in [0, 2\pi]$. These angles relate to the switching angles $\alpha_i \in [0, \pi]$ through

$$\gamma_i = \alpha_i + \frac{1 - \Delta u_i}{2}\pi\,. \tag{2}$$

Note that HWS implies that the d virtual angles $\gamma_i \in [0, \pi]$ relate to positive switching transitions, i.e., $\Delta u_i = 1$ with $\alpha_i \in [0, \pi]$, while the remaining d virtual angles $\gamma_i \in [\pi, 2\pi]$ relate to negative ones, i.e., $\Delta u_i = -1$ with $\alpha_i \in [0, \pi]$.

With the above, the proposed single optimization problem for a κ-level converter is of the form[1]

$$\begin{aligned}
\underset{\boldsymbol{\gamma}_H}{\text{minimize}} \quad & J(\boldsymbol{\gamma}_H) = \textstyle\sum_{n=5,7,\ldots} \frac{a_n^2(\boldsymbol{\gamma}_H)+b_n^2(\boldsymbol{\gamma}_H)}{n^2} \\
\text{subject to} \quad & a_1 = 0\,, \quad b_1 = \tfrac{\kappa-1}{2} m \\
& 0 \leq \gamma_1 \leq \gamma_2 \leq \ldots \leq \gamma_d \leq \pi \\
& \pi \leq \gamma_{d+1} \leq \gamma_{d+2} \leq \ldots \leq \gamma_{2d} \leq 2\pi \\
& 0 \leq u_i \leq l \;\; \forall i \in \{1, \ldots, 2d\}\,,
\end{aligned} \tag{3}$$

where $\boldsymbol{\gamma}_H = [\gamma_1 \; \gamma_2 \; \ldots \; \gamma_{2d}]^T$ and $l = \frac{\kappa-1}{2}$ is the maximum voltage level. It is worth noting that, as opposed to problem (1) where the switching sequences are predetermined, constraint $0 \leq u_i \leq l$ is required in (3) to guarantee that the levels of the resulting switching sequence are within the allowed range. For this constraint the switching angles α_i are first derived from γ_i, and are subsequently sorted in an ascending order. Following, the switch positions $u_i = \sum_{j=1}^{i} \Delta u_j$ are determined during the optimization procedure, where switching transition Δu_j corresponds to the j^{th} sorted switching angle α_j, with $j \in \{1, \ldots, 2d\}$.

III. Numerical Results

This section shows the optimization results of five- and seven-level HWS OPPs. To quantify the benefits of HWS, the case of pulse number $d = 7$ is considered. The chosen performance metric is the *relative* current TDD, defined as the normalized difference between the current TDD of HWS and QaHWS OPPs, i.e.,

$$I_{\text{TDD}}^{\text{rel}} = \frac{I_{\text{TDD,HWS}} - I_{\text{TDD,QaHWS}}}{I_{\text{TDD,QaHWS}}}\,. \tag{4}$$

The current TDD is calculated assuming a 0.25 per unit load reactance. As can be seen in Fig. 3, HWS OPPs reduce I_{TDD} over a wide range of modulation indices, achieving a reduction of up to 13% for both five- and seven-level converters. This improved performance is due to the additional degrees of freedom provided by the symmetry relaxation.

To reap the benefits of HWS OPPs, as illustrated above, their computation should be rendered possible. Fig. 4 shows the computation times of problems (1) and (3) for five- and seven-level OPPs and different pulse numbers based on an Intel(R) Xeon(R) CPU E5-2620 v3 @ 2.40 GHz. Taking five-level HWS OPPs with $d = 7$ as an example, the conventional approach requires about two days to provide the solution, whereas the proposed method requires only four hours. Similarly, the computation time of seven-level HWS OPPs with $d = 6$ is reduced from three days to only 4.5 hours, i.e., by more than 93%. Thus, it is evident that the proposed method enables the computation of OPPs with more pulses.

IV. Conclusions

This paper presented the reformulation of the optimization problem for multilevel OPPs with relaxed symmetry properties. As shown, the proposed formulation reduces the required computation time by more than 90% compared to the conventional solution method, thus enabling the capitalization on the benefits of HWS OPPs.

[1]As shown in [6], transformation (2) turns the Fourier coefficients a_n and b_n into a function of γ_i since $\Delta u_i \sin(n\alpha_i) = \sin(n\gamma_i)$ and $\Delta u_i \cos(n\alpha_i) = \cos(n\gamma_i)$ hold when n is odd.

Acknowledgment

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